POTENTIALS FOR OPTIMIZATION OF RETAINING WALLS' DESIGN

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The idea for the paper arises from the evolution of the interpretation of the soil's shearing resistance, which later reflects in the calculation of earth pressures and the design of retaining walls. There, according to the National Annex to Eurocode 7 (EC7), only the active, not the passive earth pressure, is considered. Namely, although traditionally the failure envelope of the Mohr's stress circles for soils is adopted as linear and inclined, in the scientific community, it has increasingly been treated as a non-linear function. Therefore, within the paper, their analysis and comparison is carried: a theoretical review is given for the linear and non-linear failure envelopes, after which a methodology for determination and application of non-linear shearing resistance in software is described, offering possibility for comparison of results for the amounts of active earth pressure forces (E_a) obtained by applying parameters from the interpretation of laboratory tests of fine-grained soil material with linear and non-linear failure envelopes in the design of retaining walls with certain heights.

Keywords: shearing resistance, non-linear failure envelope, earth pressure, Eurocode 7

1. INTRODUCTION

The purpose of this paper is to offer an advanced, but insufficiently applied approach for the calculation of active earth pressure on retaining walls. Namely, when interpreting the shearing resistance of soils, the theory of Mohr-Coulomb-Terzaghi is most often used, with which it is assumed that the linear function is a sufficiently accurate description of soil's failure in total domain of stresses: both below and above those applied during laboratory investigations. However, numerous tests in a wide stress domain have undoubtedly shown that the failure envelope is not linear. Thus, the behavior of the soil under a different range of normal stresses indicates doubt in the constancy of the angle of internal friction for any state of stresses for the same soil material, which is why, between the many geotechnical structures and works, the analysis of actions on retaining walls will have to undergo changes.

Namely, as there is difference in the shearing resistance parameters determined through linear and non-linear failure envelopes, the same applies to the earth pressures on the retaining walls, especially in the zone of low stresses, since such are most common behind them. In the paper, a comparison of these two theories for interpretation of shearing resistance and their influence on the active earth pressure is made.

2. REVIEW OF SOIL'S SHEARING RESISTANCE THEORIES

2.1 LINEAR MOHR-COULOMB-TERZAGHI FAILURE ENVELOPE

One of the most commonly used relations for interpreting shearing resistance τ of soil materials, defined by ϕ' – angle of shearing resistance, and c' – cohesion, in routine geotechnical practice is the Mohr-Coulomb's law, later modified by Terzaghi who separated the total normal stresses σ and pore pressures u, which is fundamental in geotechnics:

$$\tau = c' + (\sigma - u) \cdot \tan\varphi' \qquad (1)$$

However, this theory simplifies the shearing strength of soils because it assumes constant and unaffected parameters over the entire range of loads.

2.2 NONLINEAR FAILURE ENVELOPE OF HYPERBOLIC TYPE

The non-linear failure envelope [5] provides answers to some of the limitations of the linear one [6], particularly its inability to represent the behavior of soils under different stress states, especially those not applied during the test. It introduces a non-linear relationship between the normal stress and the shearing resistance, and at the same time expresses the strength only through angle of shearing resistance which changes magnitude as a function of the normal stress (i.e., variable angle and no cohesion). The general form of the non-linear failure envelope of the hyperbolic type can be expressed as:

$$\tau = \sigma'_n \cdot \tan\phi'_{\sigma'_n} \qquad (2)$$

From the stress perspective, the fracture mechanism is further complicated by the constant contact between the grains in the material, which causes the grains to move and slide under the action of normal stresses, and the fact that the local stresses in the material between the grains, which can be several times greater than the total stress of the entire material, also causes the grains to crush and break. This phenomenon is the cause of differences in the peak strength of the soil and the strength at large deformations. Taking all this into account, the previous relationship is more precisely expressed by

$$\tau = \sigma'_n \cdot \tan\left(\varphi_B + \frac{\Delta\varphi}{1+\frac{\sigma_n}{p_N}}\right) \qquad (3)$$

where:

ϕ'_B - basic friction angle, which represents the angle of shearing resistance that occurs at high stresses, when there is no change in the volume of the material, but the developed stresses lead to their breakage. When the stresses are complete and do not change magnitude, then this angle reflects the friction at constant volume ϕ'_{cv}.

$\Delta\phi'$ - is the maximum angular difference, which in soils with compactly arranged grains considers the effect of dilatancy ($\phi'_B+\Delta\phi'=\phi_0$ – initlal, maximum, angle), and reflects compactness and roundness, while in those with high percentage of flat grains, at large deformations, it expresses an imperfect arrangement of the grains relative to the shear plane, at almost "zero" normal stresses.

p_N - stress that expresses the strength of grains against crushing, which depends on the compactness, grain size distribution, mineralogical composition and grains' shape.

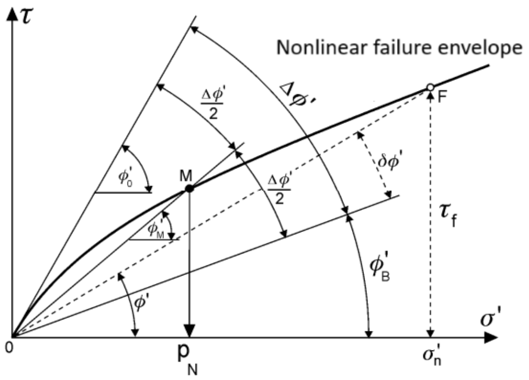

Figure 1. Parameters of a non-linear failure envelope of hyperbolic type

3. WORK METHODOLOGY AND ANALYZED CONDITIONS

The difference in the application of linear and non-linear failure envelopes will be examined through a design of reinforced concrete cantilever retaining walls, under previously defined conditions:

- Geomechanical conditions – in order to threat non-linearity for laboratory-defined material characteristics.
- Geometric conditions – different heights of the structure. The non-linearity of the failure envelope is mostly pronounced in the zone of relatively small normal stresses, like those that occur up to certain depths behind retaining walls.

The values of Ea are calculated for shearing resistance parameters determined with both strength theories. In the case of a linear failure envelope, the shearing resistance parameters (φ', c') are constant along the height of the retaining wall, so in conditions of fine-grained materials, both parameters are taken to determine the amount of active earth pressure. On the other hand, the interpretation with a non-linear failure envelope is made in the NENVE software using a hyperbolic type of curve: a variable shearing resistance angle is determined, which decreases from ϕ_0 to ϕ'_B as the normal stresses rise, while the "cohesion" does not exist. In it, the normal stresses applied in laboratory experiments are entered, as well as auxiliary angles determined geometrically from the test results [7-8]. In the output section, in addition to the parameters of the non-linear failure envelope (basic angle, angular difference, stress), the software offers a range of diagrams, and of interest to the research in question are those where the relationship between the shearing resistance angle and the normal stresses is given. As mentioned, the angle ranges from initial (maximum) value ϕ_0 to basic angle (minimum) ϕ'_B, noting also the importance of p_N. The dependence can be displayed both on a linear and semi-logarithmic scale, from which the shearing resistance angles for different stress levels can be simply readout, which is what was done. Namely, in the calculations for Ea, the shearing resistance angle varies depending on the normal stress that grows with depth. Therefore, in the calculation of Ea, in the theory of the non-linear failure envelope, the soil material is divided into fictitious layers with a thickness of 1 m. Once the normal stresses in the middle of these layers have been determined for the laboratory-derived bulk density, these values are applied to the output diagrams from NENVE in order to readout for the angles: this procedure takes into account the change in angle, which value is later assigned to the corresponding layers, and for such an amount of shearing resistance angle – the Ea is determined.

3.1 GEOMECHANICAL CONDITIONS

Within the framework of the paper, findings from analyses performed with parameters interpreted from direct shear tests on fine-grained material (in practice called "coherent") are presented. In order to "fulfill" the conditions for effective stresses and drained conditions, it is assumed that there is a drainage system with perforated pipes on the back of the wall and that the groundwater level is deep. Below are excerpts from conducted tests for the analyzed fine-grained material.

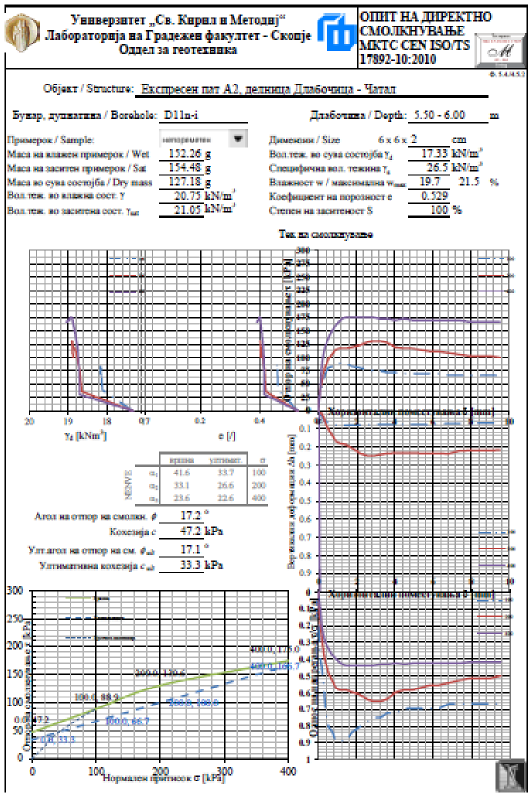

Figure 2. Parameters obtained from direct shear test conducted on fine-grained material

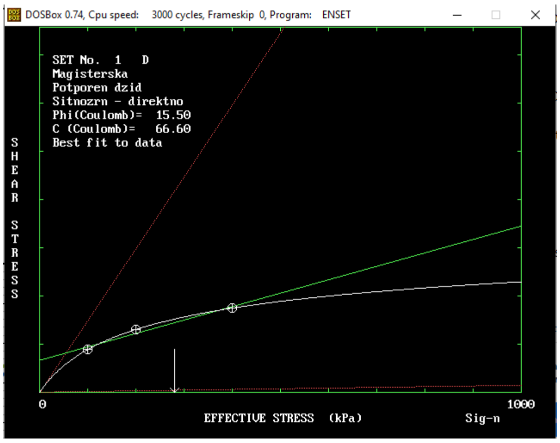

Figure 3. Interpretation with linear failure envelope in the NENVE software

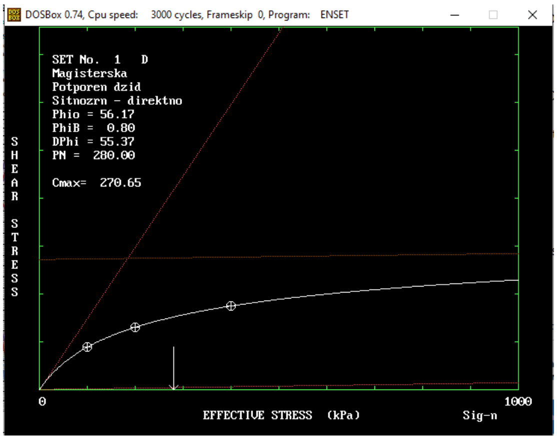

Figure 4. Parameters of non-linear failure envelope of hyperbolic type in the NENVE software

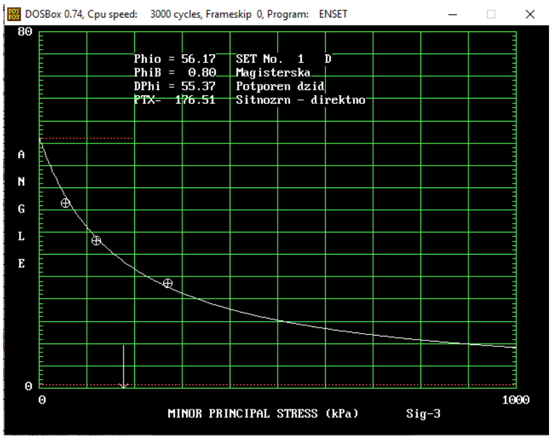

Figure 5. Relationship between angle of shearing resistance and normal stresses (NENVE)

Linear failure envelope (constant parameters)		Non-linear failure envelope of hyperbolic type (variable parameters)	
Angle of internal friction ϕ' [deg]	Cohesion c [kPa]	Initial angle of shearing resistance ϕ_0 [deg]	Mean stress p_N [kPa]
17,2	47,2	56,17	270,65

Table 1. Shearing resistance parameters of tested fine grained material interpreted with linear and non-linear failure envelopes

3.2 GEOMETRICAL CONDITIONS

Retaining walls with heights h=2-10 m, foundation widths B=0.6h and constant thickness of structural parts of 0.1h for the wall face and 0.15h for the footing, are modeled in the GEO5 software, with the soil parameters conditions given above, both for constant and variable strength parameters, including unit weight of 20,75 kN/m^3, through equilibrium boundary conditions.

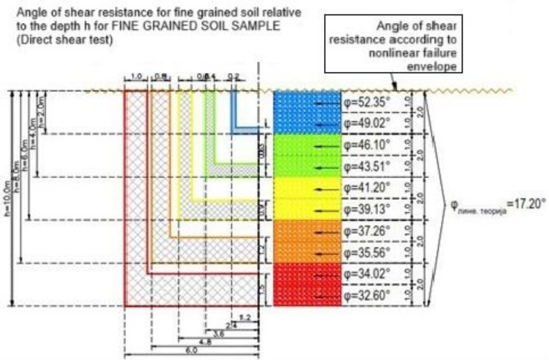

Figure 6. Geotechnical model: overview of geometrical conditions of the retaining wall and geomechanical conditions with constant and variable shearing resistance parameters

4. FINDINGS FROM NUMERICAL ANALYSES

According to the parameters determined through linear (constant shearing resistance parameters) and through non-linear failure envelope (angle of shearing resistance as a function of normal stresses), Ea are calculated. In addition, for the non-linear failure envelope, due to the height of the wall, the angle would continuously decrease to some value at a constant volume. The variable angle is applied to fictitious "layers" with a thickness of 1 m, according to the findings of NENVE (for calculated normal stresses in the middle of “layers”, angles are read from Figure 5). The corresponding angles thus calculated, depending on the depth of the "layer", are shown in Figure 6. Similarly, the value marked on the drawings as “φ-lin. theory” represents the angle of internal friction obtained from laboratory experiments, which fixed value obtained from a linear failure envelope is used together with cohesion in parallel calculations for Ea, both of them in GEO5.

So, the presented parameters are applied in the GEO 5 software module for the described cantilever retaining walls, where the active earth pressure forces are calculated according to former Macedonian standards (MKS), Design Approach 2 (DA2) of EC7 and Design Case 1 (DC1) with Resistance Factor Approach (RFA) of EC7 2025, which is the same as DA2 of EC7 [1]. The comparison of the force values for all walls, both theories and design procedures, is given in Table 2.

The failure envelopes and their influence on Ea are also reflected in the safety factors for various stability analyses, interpreted through the MKS, EC7 and EC7 2025 [8], which part, however, will be omitted here.

Para-meters inter-preted with failure envelope	Active earth pressure force [kN] at wall with height [m]				
	2m	4m	6m	8m	10m
Linear	/	5,19	39,89	123,83	259,17
Non-linear	/	/	4,56	22,90	71,32

Table 2. Forces of active earth pressure acting on retaining walls with different heights – comparison between values obtained with parameters determined through linear and non-linear failure envelope

From the comparison of the values of active earth pressure forces calculated according to DA2 of EC7, which is reflection of the so-far applied MKS, and the DC1 with RFA of EC7 2025, which is in detail given in [8], it is observed that:

- Calculations according to MKS, DA2 and DC1 give mutually literary the same results when applying linear and non-linear failure envelope;

- Values of Ea obtained with a variable angle of shearing resistance are smaller than those calculated with constant parameters (due to the presence of pronounced cohesion, a larger difference in Ea occurs).

- The increase in E_a is smaller in the case with variable parameters due to the dilatancy, and thus a larger angle of shearing resistance in a long stress range.

- from Ea values it can be seen that in this particular case, at a variable angle of shearing resistance, for heights up to 4 m, there is no need for a support/wall.

- It should also be noted that Ea does not act at vertical plane, but the software treats action at an inclined one, even though the width of the foundation is greater than H·tan(45+φ/2).

5. IMPLICATION

Among the other, the presented findings also affect the material consumption. Thus, as far as reinforcement at the retaining wall is concerned, by applying shearing resistance parameters determined through non-linear failure envelope, in addition to the fulfilled stability conditions, a significant saving in reinforcement can be achieved, which indicates that solid optimization in terms of the thicknesses of the structural elements is also possible!

For example, in the specific case, for a wall height of 10 m, when modeling the soil with shearing resistance parameters obtained through linear failure envelope, the calculations in GEO5 require about 3000 mm^2 of reinforcement, while by applying variable angle, there is almost no need for reinforcement due to the sufficient strength of the soil and assumed dimensions of the wall.

Another view of the results will be given from the aspect of the action of Ea. Namely, all previous results, analyses and comments refer to an inclined plane, which treats the plane on which Ea acts as an inclined surface at angle α=(45-φ/2), although the width of the foundation (0.6H) is greater than H·tan(45+φ/2). However, the GEO5 software package offers the possibility to manually adopt the force to act on a vertical (virtual) plane, which was done within the framework of the research, when all calculations were repeated for the action of a force on a vertical plane. Below is table for comparing the values of the active earth pressure forces calculated by applying DA2 of EC7, i.e. DC1 of EC7 2025, for parameters determined through both theories, as well as for both planes, for mid and maximum heights of the wall.

Height H [m]	6				10			
Plane	I		V		I		V	
Failure envelope	L	NL	L	NL	L	NL	L	NL
Value of Ea [kN]	39,89	4,56	/	/	259,17	71,32	/	/

Table 3. Comparison of forces of active earth pressure Ea for retaining walls with heights 6 m and 10 m achieved for inclined and vertical planes of action when applying parameters determined from linear and non-linear failure envelopes for fine grained material (Legend: 'I' stands for Inclined, 'V' stands for Vertical, 'L' stands for Linear and 'NL' stands for Non-linear).

From table 3 it can be seen that Ea on an inclined plane are many times higher than those on a vertical plane, according to the two theories, where the differences and amounts vary with increasing wall height.

Similar comparison is made also for safety coefficients [8], but it is out of the scope of the paper.

6. CONCLUSIONS

When designing geotechnical structures, typical engineering practice is to apply soil's shearing resistance parameters determined through linear interpretation of failure envelope, e.g. constant angle of internal friction and cohesion. Such can be valid for the span of loads applied during the laboratory test used for determination of shearing resistance. However, in the case of loads below and above those, and not rarely even within them, such approximation might not be satisfying. In such circumstances and for such purposes, interpretation with non-linear failure envelope is necessary and recommended. One of them is the one of hyperbolic type: it results with angle of shearing resistance which varies (decreases) as the stress increases, while there is no cohesion. This is especially important for the zones of loads below those applied during the test, which appear behind retaining walls and until certain depth at slopes and are as such important in these cases, due to mis-interpretation of shearing resistance.

The difference in the shearing resistance parameters determined through linear and non-linear failure envelopes reflects to the earth pressures on the retaining walls. The paper gave comparison of values of forces obtained at retaining walls with heights 2-10 m by applying parameters determined by these two theories for a fine grained material.

The methodology for modelling variable angle of shearing resistance along the height of the wall is described in detail.

The parameters determined by linear and non-linear failure envelope are included in the GEO 5 software for the mentioned cantilever walls, where the Ea is calculated according to MKS, DA2 of EC7 and DC1 with RFA of EC7 2025. By comparing their values, it was found that DA2 and the DC 1 result in same values of Ea, both when applying linear and non-linear failure envelope parameters. Also, the values of Ea obtained with a variable angle of shearing resistance are smaller than those calculated with constant parameters, while the increase in E_a is smaller in the case with variable parameters.

All these findings support the future application of the new generation of Eurocode 7 and stimulate the use of advanced shearing resistance theories in engineering practice in order to optimize the design of retaining walls.

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