LOCAL GEOID DETERMINATION IN WESTERN MACEDONIA BY LSMSA METHOD

This paper presents the fundamental components of the Least Squares Modification of the Stokes integral with Additive corrections (LSMSA) to geoid determination. This method also known as the KTH method is theoretically described, then it is applied to the western part of North Macedonia. In this regard, input data is firstly explained, and then various geoid models are created in the test area. Finally, all geoid models are evaluated against the ground truth to get a final geoid model. Numerical results indicate that despite of limited terrestrial gravity data, a precise geoid model by LSMSA method is computed in the test area.

Key words: additive corrections, digital elevation model, geopotential model, gravity anomalies, KTH method

1. INTRODUCTION

Along with Earth's topographic surface and ellipsoid, the geoid is one of the three main surfaces in geodesy. The geoid is the reference surface for physical heights. It can be described with many definitions and one of them is: "The geoid is the equipotential surface of the Earth's gravity field that most closely coincides with the undisturbed mean sea level (and its continuation through the continents). Disturbances are caused by ocean tides, streams, winds, variations in salinity and temperature, of the order of ± 2 m" [1].

The geoid is a physical surface, and its best approximation is the mean sea level. The geoid model plays an essential role in many engineering and scientific applications, such as:

- Reference surface for leveling,
- Vertical datum for orthometric heights,
- Transformation of ellipsoidal (geometrical) to orthometric heights (physical),
- Studies of the Earth's interior and ocean,
- Research for deposits of oil and gas etc.

One of the main uses of geoid model is for transformation of ellipsoidal heights obtained by *GNSS (Global Navigation Satellite System)* to physically meaningful orthometric heights by,

$$H = h - N \qquad (1)$$

The geoid undulations *N* are defining the geoid model and they can be interpreted as the differences between the orthometric height (measured along the plumb line from the geoid) and ellipsoidal height (measured along the ellipsoidal normal line from reference ellipsoid). The type of measurement method for obtaining height differences which consists of measuring ellipsoidal heights *h* using *GNSS* and geoid undulations *N* from geoid model is called *GNSS levelling.* This method is intended as a replacement to the classical levelling using a pair of rods and it could save time and cost (Fig. 1).

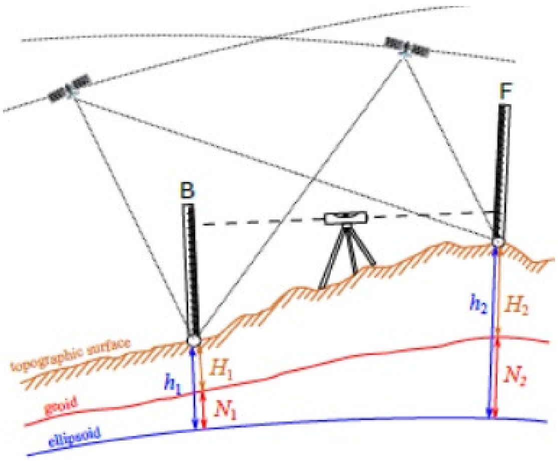

Figure 1. Determination of height differences using classical levelling and GNSS levelling

The main aim of this paper is to obtain a local gravimetric geoid model as accurate as possible. To the best of our knowledge, there has not been any precise gravimetric geoid model in our test area.

The methodology and input data for determining a geoid model will be explained in the next sections. Then, numerical applications will be realized in the western part of Makedonia. Lastly, comparison results will be discussed in the final section.

2. BASIC CHARACTERISTICS OF THE LSMSA METHOD

Least Squares Modification of Stokes formula with Additive corrections (LSMSA) is developed at the Royal Institute of Technology in Sweden by L. E. Sjöberg [9]. It is one of frequently used methods for determination of a geoid model [1,6,7,13]. In this approach, terrestrial free-air anomalies and global geopotential models are combined for calculation of the approximate geoid undulations. Then, the additive corrections are computed separately and added to the previously computed approximate undulations to get the final geoid model.

Let's start with the fundamental equation in the physical geodesy, which is the Stokes formula for determining geoid undulations:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi) \Delta g d\sigma \qquad (2)$$

This is the Stokes kernel to determine the undulation on a sphere as a body that approximates the Earth, but the Earth is not a sphere. Thus, Sjöberg [8,12] did some modifications and the eq. (2) is rewritten as:

$$\tilde{N} = \frac{R}{4\pi\gamma} \iint_{\sigma 0} S_L(\psi) \Delta g d\sigma + \frac{R}{2\gamma} \sum_{n=2}^{M} b_n \Delta g_n \qquad (3)$$

where Δg is the free-air anomaly, $\sigma 0$ is the cap with a spherical radius ψ_o, S_L is modified Stokes kernel, Δg_n is the free-air anomaly from GGM (Global Geopotential Models).

Since the integration should be done to the whole sphere according to Stokes formula, we need the gravity anomalies for whole Earth. Eventually, we only have the gravity measurements for specific target area, while the other gravity anomalies are gathered from the GGMs. Therefore, the modification of the original Stokes kernel was needed.

Considering assumptions above, the final geoid model is given as follows:

$$N = \tilde{N} + \delta N_{top}^{comb} + \delta N_{DWC} + \delta N_{atm} + \delta N_{ell} \qquad (4)$$

where $\tilde{N}$ represents the approximate geoid undulations (eq. 3), δN_{top}^{comb} represents the combined topographic correction, δN_{DWC} represents the downward continuation correction, δN_{atm} is the atmospheric correction and δN_{ell} is the ellipsoidal correction.

The combined topographic correction [11] is carried out by the orthometric height of the point,

$$\delta N_{comb}^{Top} = -\frac{2\pi G \rho H^2}{\gamma}\left(1 + \frac{2H}{3R}\right) \qquad (2)$$

This correction has the largest impact of all four corrections and its value can be in the range of decimeters. Moreover, interested reader can exploit actual topographic density instead of the standard density (2670 kg/m^3) to gain more precise results [e.g. 1].

The downward continuation correction [10] of the gravity anomalies can be expressed as,

$$\delta N_{dwc}^{(1)} = \frac{\Delta g_P}{\gamma} H_P + 3\frac{\tilde{N}}{r_p} H_P - \frac{1}{2\gamma}\frac{\partial \Delta g}{\partial r}\Big|_P H_P^2 \tag{3}$$

$$\delta N_{dwc}^{L1,Far} = \frac{R}{2\gamma}\sum_{n=2}^{M} b_n \left[\left(\frac{R}{r_P}\right)^{n+2} - 1\right]\Delta g_n \tag{4}$$

$$\delta N_{dwc}^{L2} = \frac{R}{4\pi\gamma}\iint_{\sigma_o} S^L(\psi)\left[\frac{\partial \Delta g}{\partial r}\Big|_Q (H_P - H_Q)\right] d\sigma_o \tag{5}$$

The value of this correction is in the range of several centimeters.

The atmospheric correction is computed by [8]:

$$\delta N_{comb}^{Atm} = -\frac{GR\rho^a}{\gamma}\iint_{\sigma 0} S^L(\psi) H_P d\sigma_o \tag{6}$$

The ellipsoidal correction [3] due to the Earth's approximation with a sphere, can be simply calculated by,

$$\delta N_{ell} \approx [(0.0036 - 0.0109 sin^2\varphi)\Delta g + 0.0050\tilde{N} cos^2\varphi] Q_0^L \tag{7}$$

The atmospheric and ellipsoidal corrections are in the range of a few millimeters.

We can conclude several aspects about the calculations:

- The approximate geoid undulations are dependent of the terrestrial gravity anomalies and GGM,
- Topographic correction depends on the digital elevation model (DEM),
- Downward correction depends on DEM GGM, terrestrial gravity anomalies,
- Atmospheric correction depends on DEM,
- Ellipsoidal correction is dependable on the approximate geoid undulations and free-air anomalies.

3. STUDY AREA FOR LOCAL GEOID

The study area for determination of geoid model is located in the western part of the Republic of North Macedonia. The target area is bounded by the points in Tab. 1.

The study area is characterized with a dynamic topography. The minimum, maximum, and average heights are 594, 2102, and 1264 m, respectively.

Table 1: Target area boundaries

Point	ϕ	λ
Southwest	41°18′18″	20°39′18″
Northwest	41°38′42″	20°39′18″
Southeast	41°18′18″	20°59′42″
Northeast	41°38′42″	20°59′42″

For the determination of a local geoid model, three datasets are needed (terrestrial gravity anomalies, GGM, DEM). Some of the data are accessed with the permission from state authorities, namely the National Agency for Cadastre which provides the gravity surveys, GNSS and high precision levelling data. GGM and DEM are publicly available via the webpages ICGEM [4] and Earth Explorer [14], respectively.

The total number of gravity points over the target area is 165, which derivates the terrestrial free-air gravity anomalies (Fig. 2).

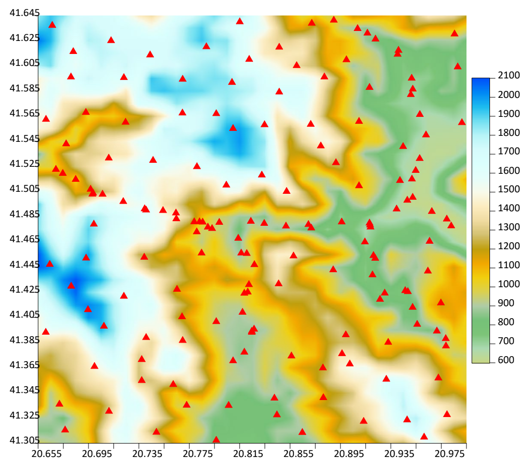

Figure 2: Distribution of terrestrial gravity points in the target area

According to the boundaries, the length of the area is the same as the width of the area, which is 20'24'' (total area equals to 1000 km^2). The density of the gravity data is 1 point per 6 km^2, which is better than a couple of studies showing that good density is approximately 1 point per 10 km^2. From Fig. 2, the gravity data is regularly distributed around the target area.

On the other hand, we have 46 GNSS-levelling benchmarks which can help us derive geometrical geoid undulations using the eq. (1). GNSS-levelling data will be used for fitting the gravimetric geoid model to local vertical datum. In the following sub-sections, GGMs and DEMs which are employed in this study will be treated.

3.1 Global Geopotential Models

Global geopotential models represent the long wavelength information of the Earth's gravity field. The models are represented by a series of spherical harmonics with different degree (n) and order (m). While the maximum harmonic expansion was around 30-40 degrees in the past, today the most detailed models are produced by degree expansion up to 5500 [4]. They are created by the different satellite missions (GRACE, GOCE, CHAMP), terrestrial gravity data as well as topography.

When a geoid model is calculated, the question of which GGM to use always arise. In this situation, there are no restraints on which model to use, but in order to get more accurate model, the GGM that is intended for use should be evaluated by the ground truth in form of gravity anomalies or geometrical geoid undulations. The LSMSA method mostly uses satellite-only GGM. Thus, in our case, the evaluation is carried out on eight satellite-only GGMs (Tab. 2). The choice of these eight models is done by the most recent releases and derivation from two missions such as GRACE, GOCE or GRACE+GOCE models.

Table 2: GGMs used in this study

Name	Year	Max. degree of expansion
Tongji-Grace02k	2018	180
HUST-Grace2016s	2016	160
GOSG02S	2023	300
GO_CONS_GCF_2_TIM_R6	2019	300
GOSG01S	2018	220
WHU-SWPU-GOGR2022S	2023	300
Tongji-GMMG2021S	2022	300
ITU_GGC16	2016	280

From Tab. 2, the maximal expansion for the GGMs is ranging from 160 to 300 degrees. For the validation of the GGMs, we need the gravimetric geoid undulations from the ICGEM calculation center [4] using full expansion for each GGM and the geometrical geoid undulations derived from GNSS-levelling data. The comparison results are listed in Tab. 3.

Table 3: Validation of GGMs on GNSS-levelling points

Name	Absolute validation [cm]		
	Min.	Max.	RMS
Tongji-Grace02k	-105.48	2.92	30.06
HUST-Grace2016s	-174.38	-82.78	25.22
GOSG02S	-39.88	39.61	21.20
GO_CONS_GCF_2_TIM_R6	-41.38	41.02	22.08
GOSG01S	-62.48	15.88	21.60
WHU-SWPU-GOGR2022S	-40.68	38.71	21.21
Tongji-GMMG2021S	-48.18	32.01	21.30
ITU_GGC16	-59.48	23.82	22.21

From Tab. 3, we can conclude which GGM is the best suitable for use in the computation of the geoid model based on the RMS (root mean square error) of the differences between gravimetric (GGM) and geometric (GNSS derived) geoid undulations. Based on the table, we can see that the values are similar to each other. The model that gives smallest RMS is GOSG02S. The worst one is Tongji-Grace02k with RMS of 30.06 cm. In order to eliminate possibly systematic bias in the comparison, we employ corrector surfaces [5, 7]. In this case, 4 parameter is used in this study and results are given in Tab. 4.

Table 4: Fitting of GGMs to GNSS points (4 par. fit)

Name	4 parameter fit [cm]		
	Min.	Max.	RMS
Tongji-Grace02k	-17.15	20.39	6.44
HUST-Grace2016s	-17.10	20.69	6.49
GOSG02S	-17.20	20.37	6.45
GO_CONS_GCF_2_TIM_R6	-17.20	20.26	6.41
GOSG01S	-17.20	20.29	6.39
WHU-SWPU-GOGR2022S	-17.18	20.40	6.44
Tongji-GMMG2021S	-17.21	20.70	6.51
ITU_GGC16	-17.18	20.26	6.41

From Tab. 4, we can see that all GGMs give precise results comparing them with GNSS-levelling points. The GGM that derives smallest RMS is GOSG01S.

3.2 Digital Elevation Models

Digital elevation model (DEM) is the representation of the Earth’s topography as well as the short wavelength variations in the Earth’s gravity field. In our case, two mostly used DEMs are employed: SRTM (Shuttle Radar Topography Mission) and ASTER (Advanced Space-borne Thermal Emission and Reflection Radiometer). The technical specifications of the DEMs are available in Tab. 5

Table 5: SRTM and ASTER data [14,15]

Type of info	SRTM	ASTER
Institution	NASA	NASA, METI
Resolution	1”	1”
Horizontal datum	WGS84	WGS84
Vertical datum	EGM96	EGM96

Before using any DEM, it should be evaluated with the ground truth, which means comparison with the 46 levelling points in our case. We need to check the differences between the orthometric heights derived from the DEMs and those measured with classical levelling. The results are shown in the Tab. 6 using the 4-parameter fit because of the different vertical datums between the DEMs (EGM96) and the levelling points (mean sea level).

Table 6: Comparison results of DEMs

Model	Absolute validation – 4 par. fit [cm]		
	Min.	Max.	RMS
SRTM	-131.02	134.7	61.21
ASTER	-131.53	134.31	61.22

The numerical results present same agreement after 4-parameter fitting between both DEMs and levelling points.

4. PRACTICAL COMPUTATION OF LOCAL GEOID MODEL

The input data is gathered in form of terrestrial free-air anomalies, global geopotential models and digital terrain models. In geoid modelling studies, we need the data for the target area and surrounding area. The data area will be within these boundaries in Tab. 7. The data area is 1 degree wider on each side from the boundaries of the target area.

Table 7: Boundaries of data area

Point	ϕ	λ
Southwest	40°18′18″	19°39′18″
Northwest	42°38′42″	19°39′18″
Southeast	40°18′18″	21°59′42″
Northeast	42°38′42″	21°59′42″

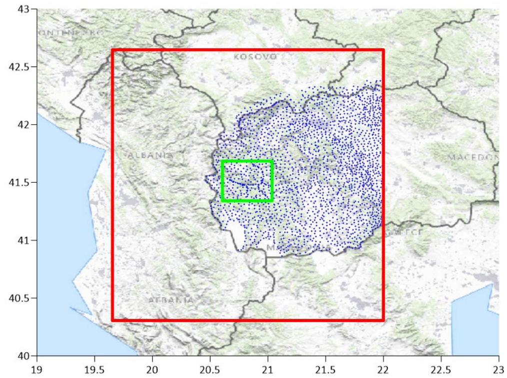

Figure 3: Data and target area

Fig. 3 shows the data area marked with red line, target area with green line, and terrestrial free-air anomalies marked with blue color. Whereas the data coverage is well on the eastern side of the target area, the other parts will be fulfilled with anomalies derived from EGM2008, because it is difficult to obtain the gravity data from the neighboring countries.

For the practical computation of geoid model, LSMSSOFT will be used in this study [1]. The LSMSSOFT starts with three files:

- First one is the GGM file in GFC format that is obtained from ICGEM web page. This file contains the spherical harmonic coefficients up the maximum expansion degree for the selected GGM;
- Second one is the free-air gravity anomaly file. For this one, we have to interpolate gravity anomalies on grid centers using the Bjerhammer rule and nearest neighbour technique. The grid resolution plays an important role since higher resolution is better approximation. Therefore, the resolution of 36 arc-second which equals to 0.01 arc-degrees was chosen in our case.
- Third one is the elevation file consisting of the orthometric heights from DEM. Averaged heights on the grid centers were created for both DEMs. Both the resolution and coverage of elevation data must match the gravity anomaly data.

The LSMSSOFT coded with C++ programming language can be executed on Linux platform

[2]. Several parameters can be chosen arbitrarily to create a geoid model:

- Firstly, we need to determine the maximum degree of expansion for the GGM (e. g. 100, 150, 200, 250, 300)-GOSG02S;
- Secondly, we need to choose the spherical integration cap size (i. e. 0.5 or 1 degree);
- Finally, variance of terrestrial gravity anomalies (e. g. 1, 2, ..., 30).

Considering these parameters, a wide range of geoid models can be created, and one must be chosen among them. The geoid model giving the smallest RMS is chosen as final geoid model, comparing with the geometric geoid derived from GNSS-leveling points. In our case, the final parameters of maximum expansion degree, cap size and variance of terrestrial gravity anomalies are 300, 1 degree, and 30 mgal^2, respectively.

Table 6: Comparison of gravimetric and geometric geoid models

	Absolute comparison [cm]		
	Min.	Max.	RMS
No fit	56.11	104.26	10.92
4 par. fit	-25.66	11.11	6.16

After comparing the gravimetric and geometric geoid models, we can conclude that the improvement of geoid model using fitting surface is significant from 21.20 cm (Tab. 3) to 10.92 cm (Tab. 7).

Furthermore, after using 4 parameter fitting surface, we can see a remarkable improvement from 10.92 to 6.16 cm. Finally, the hybrid geoid model (gravimetric geoid is fitted to the geometric geoid) is portrayed in Fig. 4.

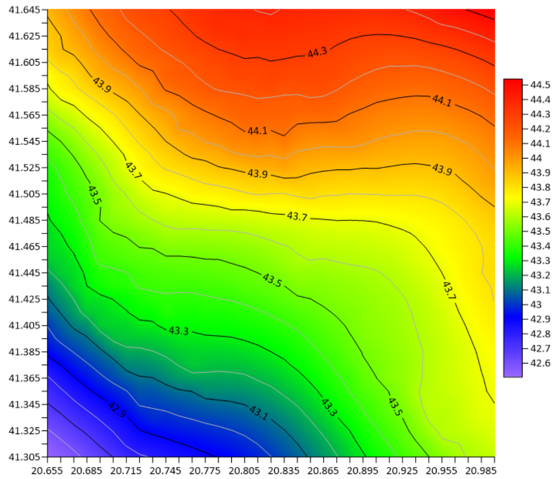

Figure 4: Hybrid geoid model by LSMSA

5. CONCLUSIONS

Based on the presented analysis in the previous sections, the following conclusions can be drawn:

- The more gravity data used in the calculation gives the better results. The aspiration should be to have at least 1 point per 4 km^2.
- The recent satellite GGMs yields the precise results.
- Better and improved DEMs should be included.
- Dozens of geoid models are calculated changing the input parameters.
- The gravimetric geoid model is checked with geometric geoid for the final solution.
- Hybrid geoid model created by parametric surface gives systematic error free model which is usable directly in the test region.

REFERENCES

[1] **Abbak, R. A. (2020).** Effect of a high-resolution global crustal model on gravimetric geoid determination: a case study in a mountainous region. Studia geophysica et geodaetica, *64*, 436-451.

[2] **Abbak R. A. and Üstün A. (2015).** A software package for computing a regional gravimetric geoid model by the KTH method. Earth Science Informatics 2015 Vol. 8 Pages 255-265. **DOI: 10.1007/s12145-014-0149-3**

[3] **Ellmann A. and Sjöberg L. (2004).** Ellipsoidal correction for the modified Stokes formula. Boll Geod Sci Aff 2004 Vol. 63.

[4] **ICGEM (2024).** International Centre for Global Earth Models https://icgem.gfz-potsdam.de/home, accessed date: November 21st, 2024

[5] **Kotsakis, C. and Sideris M. G. (1999).** On the adjustment of combined GPS/levelling/geoid networks. Journal of Geodesy 73(8), 412–421. **DOI: 10.1007/s001900050261**

[6] **Pa'suya, M. F., Din, A. H. M., Yusoff, M. Y. M., Abbak, R. A., & Hamden, M. H. (2021).** Refinement of gravimetric geoid model by incorporating terrestrial, marine, and airborne gravity using KTH method. Arabian journal of geosciences, 14, 1-19.

[7] **Pa'suya, M. F., Md Din, A. H., Abbak, R. A., Hamden, M. H., Yazid, N. M., Aziz, M. A. C., & Samad, M. A. A. (2022).** Hybrid geoid model over peninsular Malaysia (PMHG2020) using two approaches. Studia Geophysica & Geodaetica, 66(2022), 98—123,

[8] **Sjöberg L. E. (1999).** The IAG approach to the atmospheric geoid correction in Stokes' formula and a new strategy. Journal of Geodesy 1999

Vol. 73 Issue 7 Pages 362-366. **DOI: 10.1007/s001900050254**

[9] **Sjöberg L. E. (2003).** A general model for modifying Stokes' formula and its least-squares solution. Journal of Geodesy 2003 Vol. 77 Issue 7 Pages 459-464. **DOI: 10.1007/s00190-003-0346-1**

[10] **Sjöberg L. E. (2003).** A solution to the downward continuation effect on the geoid determined by Stokes' formula. Journal of Geodesy 2003 Vol. 77 Issue 1 Pages 94-100. **DOI: 10.1007/s00190-002-0306-1**

[11] **Sjöberg, L. E. (2007).** The topographic bias by analytical continuation in physical geodesy. *Journal of Geodesy*, *81*(5), 345-350.

[12] **Sjöberg L. E. and Bagherbandi M. (2017).** Gravity Inversion and Integration, Theory and Applications in Geodesy and Geophysics. Springer**. DOI: 10.1007/978-3-319-50298-4**

[13] **Krdžalić, D., & Abbak, R. A. (2023).** A precise geoid model of Bosnia and Herzegovina by the KTH method and its validation. Survey Review, 55(393), 513-523.

[14] **USGS (2024).** United States Geological Survey**,** https://earthexplorer.usgs.gov, accessed date: November 21st, 2024

[15] **Üstün, A., Abbak, R. A., & Zeray Öztürk, E. (2018).** Height biases of SRTM DEM related to EGM96: from a global perspective to regional practice. *Survey Review*, *50*(358), 26-35.