BUCKLING BEHAVIOR OF FUNCTIONALLY GRADED POROUS PLATE WITH PARABOLIC VARIABLE THICKNESS LAY ON PASTERNAK FOUNDATION: A MINI-STUDY

This article describes the buckling load of functionally graded porous plate with parabolic variable thickness (FGP-PVT) installed on Pasternak foundations. This construction also makes use of the sinusoidal porous distribution. With springer stiffness (k_1) and shear stiffness (k_2) as functions of the deflection and its Laplacian, the Pasternak foundation is a two-parameter model that depicts the foundation reaction. It is expected that the outcomes will offer information for the design of FGP-PVT plates in real-world engineering applications.

Keywords: buckling load, functionally graded porous plate, parabolic variable thickness, Pasternak foundation.

1. INTRODUCTION

Materials have always played a vital role in human existence. Certain properties result in particular uses. Materials for aircraft and aerospace applications have exceptional strength and low weight. When it comes to their capacity to possess novel characteristics and exceptional qualities that conventional materials cannot match, functionally graded materials (FGM) represent a wholly original idea [1]. Additive manufacturing is an efficient way to fabricate it [2, 3]. FGM structures must have mechanical performance, and it is important to look into how porosity affects this performance [4, 5]. The work [6] presented an effective approach to investigating the buckling and post-buckling behavior of porous functionally graded plates. The main objective of the analysis was to use a high-order continuity based on the asymptotic numerical method with the finite element method for nonlinear behaviors of a porous functionally graded material plate with different porosity distributions under various types of transverse loads. A novel quasi-3D hyperbolic theory was presented for the free vibration analysis of functionally graded porous plates resting on elastic foundations by dividing transverse displacement into bending, shear, and thickness stretching parts as in [7]. The elastic foundation could be chosen as Winkler, Pasternak or Kerr foundation. Three different patterns of porosity distributions were considered. A Galerkin method was developed for the solution of the eigenvalue problem of this quasi-3D hyperbolic plate model. The bending and free vibration of porous functionally graded beams resting on elastic foundations were analyzed in [8]. The material features of the beam were assumed to vary continuously through the thickness according to the volume fraction of components. The foundation medium was also considered to be linear, homogeneous, and isotropic and modeled using the Winkler-Pasternak law. The hyperbolic shear deformation theory was applied for the kinematic relations, and the equations of motion were obtained using Hamilton’s principle. The aim of the work [9] was to establish two-dimensional and quasi-three-dimensional shear deformation theories that could model the free vibration of functionally graded plates resting on elastic foundations using a new shear strain shape function. The proposed theories had a novel displacement field that included undetermined integral terms and contained fewer unknowns, taking into account the effects of both transverse shear and thickness stretching. The mechanical properties of the plates were assumed to vary through the thickness according to a power law distribution in terms of the volume fractions of the constituents. The elastic foundation parameters were introduced in the present formulation by following the Pasternak mathematical model. Hamilton's principle was employed to determine the equations of motion. The closed-form solutions were derived by using Navier's method, and then fundamental frequencies were obtained by solving the results of eigenvalue problems. The authors [10] gave a summary of the analytical and numerical techniques used to determine plates with functionally graded material properties that are supported by an elastic foundation. The finite elements method was used to obtain the numerical results, which were related to post-bifurcation problems of thermally loaded plates. The first-order shear deformation theory had been employed. In numerical calculations, they had used a new 16-node plate element, free of problems related to shear locking. The article [11] studied the effect of porosity distribution on the static and buckling responses of a functionally graded porous plate with all its four edges simply supported and subjected to a transverse load. The plate’s displacement field was approximated based on an inverse hyperbolic shear deformation theory involving five variables. A numerical scheme for buckling analysis of a functionally graded circular plate (FGCP) subjected to uniform radial compression, including shear deformation, rested on a Pasternak elastic foundation was presented in [12]. The linear and quadratic thickness variation patterns with various boundary conditions were considered. A modified Euler-Lagrange equation was achieved and then solved by converting the differential equation to a nonlinear algebraic system of equations. Also, based on traction-free surface without using shear correction factor, a new approach by considering shear deformation for buckling analysis of FGCP rested on elastic foundation was carried out. The paper [13] presented a unified solution for free vibration analysis of thick functionally graded porous graphene platelet-reinforced composite cylindrical shells embedded in elastic foundations. The three-dimensional theory of shell theory was introduced for theoretical formulation. The Rayleigh-Ritz method in conjugation with the artificial spring technique was employed, where the arbitrary boundary conditions could be conveniently obtained. A unified solution that comprises six different displacement functions was developed. Several other similar studies can be listed here [14-22]. The study [14] was to further expand the ES-MITC3 for analyzing the buckling characteristics of functionally graded porous variable-thickness plates with sinusoidal porous distribution. The ES-MITC3 was developed to improve the accuracy of classical triangular elements (Q3) and overcome the locking phenomenon while still ensuring flexibility in discretizing the structural domain of the Q3. The bending responses of porous functionally graded thick rectangular plates were investigated in [15] according to a high-order shear deformation theory. Both the effects of shear strain and normal deformation were included in the present theory, so it did not need any shear correction factor. The equilibrium equations according to the porous functionally graded plates were derived. The solution to the problem was derived by using Navier's technique. In the investigation [16], the buckling behaviours of porous double-layered functionally graded nanoplates in a hygrothermal environment were presented. The nonlocal strain gradient theory with two material scale parameters was developed to examine buckling behaviour much more accurately. Based on the new first-order shear deformation theory, the equations of equilibrium were obtained from the principle of minimum potential energy. To simplify the equations of equilibrium and remove the bending-extension coupling, the buckling behaviours of FG nanoplates were investigated based on the physical neutral surface concept. The paper [17] introduced a simple quasi-3D theory with Reddy shear function and a new trigonometric shear function to conduct free vibration analysis of the functionally graded plates resting on Winkler/Pasternak/Kerr elastic foundation. The proposed transverse shear strain functions satisfied the stress-free boundary conditions on the surfaces of the functionally graded plate without using any shear correction factors. The governing differential equation and boundary conditions were derived based on Hamilton's principle and the Winkler/Pasternak/Kerr type mathematical model. Vibration analysis of a functionally graded rectangular plate resting on a two-parameter elastic foundation was presented in [18]. The displacement field based on the third-order shear deformation plate theory was used. By considering the in-plane displacement components of an arbitrary material point on the mid-plane of the plate and using Hamilton's principle, the governing equations of motion were obtained, which are five highly coupled partial differential equations. An analytical approach was employed to decouple these partial differential equations. The decoupled equations of a functionally graded rectangular plate resting on an elastic foundation were solved analytically for Levy type boundary conditions. The new numerical procedure for functionally graded skew plates in a thermal environment was presented in the study [19] based on the C0-form of the novel third-order shear deformation theory. Without the shear correction factor, the theory was also taking the desirable properties and advantages of the third-order shear deformation theory. The author assumed that the uniform distribution of temperature was embedded across the thickness of the structure. Both the rule of mixture and the micromechanics approaches were considered to describe the variation of material compositions across the thickness. For thermo-mechanical analysis of functionally graded sandwich plates supported by a two-parameter (Pasternak model) elastic foundation, a refined quasi-three-dimensional shear deformation theory was developed in the paper [20]. Unlike the other higher-order theories, the number of unknowns and governing equations of this theory was only four against six or more unknown displacement functions used in the corresponding ones. Furthermore, the theory took into account the stretching effect due to its quasi-three-dimensional nature. The boundary conditions in the top and bottom surfaces of the functionally graded plate were satisfied, and no correction factor was required. The paper [22] presented an analytical investigation of the vibrations of porous functionally graded carbon nanotube-reinforced composite plates resting on elastic foundations. The plates were reinforced with randomly oriented straight carbon nanotubes, featuring four distinct reinforcement distribution patterns along the thickness. Material properties of the structure were graded along the thickness, with both symmetric and asymmetric porosity distributions considered. Utilizing high-order shear deformation theory, the equations of motion were derived from Hamilton's energy principle, and Navier's method was employed for the solutions. A physics-informed neural network method based on a two-network strategy was introduced in [23] to address the bending problem of thin plates with variable stiffness resting on an elastic foundation. The problem was governed by a fourth-order partial differential equation (PDE), and the use of a one-network strategy to solve the PDE directly might lead to convergence issues due to singular points. Following the principles of Kirchhoff plate theory, the governing PDE was equivalently transformed into four second-order PDEs. A two-network strategy was employed for solution. The authors presented numerical examples under various load conditions, plate geometries, foundation types, thicknesses, and material properties. The obtained results were validated against finite element method solutions and literature. The study [24] aimed to investigate the method of fundamental solution to the thin plate resting on the elastic foundation subjected to in-plane forces under either static or dynamic load. The fundamental solutions with Bessel's functions were derived in both static and dynamic cases. According to the principle of superposition, the boundary conditions were satisfied at collocation points in terms of densities of concentrated force at source points outside the domain. A double-source algorithm and a single-source algorithm were proposed to deal with fourth-order partial differential equations. Based on the symplectic superposition method, the T-shaped thin plate on the Winkler elastic foundation was divided into four sub-plates and solved by using the symplectic eigen expansion method, and the modes and frequencies were studied. The method began directly with the fundamental equations and a rigorous mathematical derivation without assuming the form of the solution beforehand. The approach helped circumvent the drawbacks associated with traditional semi-inverse solution methods. In addition, the theoretical calculation model and finite element analysis model of T-shaped thin plates on elastic foundation were established by using Mathematic software and ABAQUS software in paper [25]. In the research [26], the nonlinear dynamics of a clamped circular composite plate placed on a softening elastic foundation under rapid thermal loading was investigated. In the situation, based on the amount of temperature supplied to the structure and the coefficients of softening elastic foundation, two instabilities might happen one after the other. The structure would thermally buckle and deform dynamically if the applied temperature exceeded a critical level. If the softening coefficient of the elastic foundation was critical, the structure would completely lose its stability after a certain deformation range. A polymer containing graphene platelets made up the system. Based on various functions, the volume fraction of fillers varied along the thickness. The system's nonlinear dynamic equations were obtained by applying Hamilton's principle and the Von-Kármán theory. The transient heat conduction equation was solved by the cubic B-spline collocation and Crank-Nicolson procedures. A nonlinear numerical analysis regarding a functionally graded carbon nanotube-reinforced composite plate on an elastic foundation was carried out in [27]. The large deflection bending problem was formulated according to the von Kármán nonlinear theory and the hierarchical model. Additionally, the 2D natural element method—which demonstrated a high degree of accuracy even for coarse grid—was used to solve it numerically. The load increment scheme and the Newton-Raphson iteration method were combined to iteratively calculate the derived nonlinear matrix equations. The authors investigated impact analysis of functionally graded graphene nanoplatelets reinforced composite plates with arbitrary boundary conditions and resting on Winkler-Pasternak elastic foundations as in [28]. The element-free improved moving least-squares Ritz approach and the higher-order shear deformation theory were used to develop the theoretical formulation. Target plates were set to have uniform and functionally graded graphene nanoplatelet distributions throughout their thickness. The modified Halpin-Tsai model was considered to calculate the effective Young's modulus, yet the rule of mixture was used to calculate the effective Poisson's ratio and mass density. The modified nonlinear Hertz contact law was adopted to describe the contact force between the spherical impactor and target plates. The elastic buckling problem of a thin skew isotropic plate under in-plane loading resting on the Pasternak elastic foundation was numerically solved in [29] using the extended Kantorovich method (EKM). EKM had never been applied to this problem before. An investigation of the EKM accuracy and convergence was conducted. Formulations were based on classical plate theory. Using the variational calculus expressed in an oblique coordinate system, stability equations and boundary conditions terms were obtained from the principle of the minimum total potential energy. The resulting two sets of ordinary differential equations were solved numerically using the Chebfun package in Matlab software. The article [30] investigated the bending response performances of the magneto-electro-elastic laminated plates supported by the Winkler foundation or the elastic half-space under transverse mechanical loading. Assuming that the foundation was not electrically and magnetically conductive, the magneto-electro-elastic laminated plate and the elastic half-space were simulated using the scaled boundary finite element method based on the 3D theory of elasticity. Only the in-plane of the magneto-electro-elastic laminated plate or the boundary of the elastic half-space needs to be discretized, reducing the spatial dimension by one. This was due to the fact that the generalized displacement involving the electric potential, magnetic potential, and elastic displacement was considered as the nodal degree of freedom for the magneto-electro-elastic laminated plates in the scaled boundary finite element method model. Furthermore, the governing equations could be solved analytically in the radial direction of the scaled coordinate system, which made the scaled boundary finite element method particularly suitable for simulating the elastic half-space. The global stiffness coupling governing equation for the Winkler foundation-plate system was directly derived from the 3D elasticity equations of the magneto-electro-elastic laminated plate. This equation took into account the interaction between the Winkler foundation and the magneto-electro-elastic laminated plate and assumed that the foundation reactions were proportionate to the transverse displacements of the plate structure. However, the entire domain was divided into three sub-domains for the magneto-electro-elastic laminated plate-half-space system: the scaled boundary finite element method, the magneto-electro-elastic laminated plate structure, and the near and semi-infinite far foundation systems. This allowed for the determination of the stiffness matrix of each sub-domain. Therefore, the global stiffness equation of the plate-half-space system could be assembled at the same nodes using the degree of freedom matching principle. The paper [31] dealt with the impacts of temperature and moisture on the bending behavior of functionally graded porous plates resting on elastic foundations. The impacts of transverse shear deformation as well as the transverse normal strain were taken into consideration. The number of unknown functions involved here was only five, as opposed to six or more in the case of other shear and normal deformation theories. The effects due to side-to-thickness ratio, aspect ratio, thermal and moisture loads, porosity factor, and elastic foundation parameters, as well as the volume fraction distribution on the functionally graded porous plates, were investigated. The study conducted in [32] examined the vibration of orthotropic plates with bidirectional exponential grade resting on a two-parameter elastic foundation. The Pasternak elastic foundation model was used as a two-parameter foundation model. The heterogeneity of the orthotropic exponentially changed depending on the axial and thickness coordinates. The motion equation was derived based on the classical plate theory and solved by using the Galerkin method. A comparison with earlier research was done in order to validate the findings. The effects of the two-parameter elastic foundations, material gradient, and orthotropy on the dimensional frequency parameters were examined. The local buckling behavior of long rectangular plates supported by tensionless elastic Winkler foundations and subjected to a combination of uniform longitudinal uniaxial compressive and uniform in-plane shear loads was analytically solved using a mathematical approach in the paper [33]. Fitted formulas were derived for plates with clamped edges and simplified supported edges. Two examples were given to demonstrate the application of the current method: one was a plate on tensionless spring foundations, and the other was the contact between the steel sheet and elastic solid foundation, etc.

The FGP-PVT plate formulation is taken into consideration to start this investigation, particularly with regard to porosity appearance. The article concludes with some notes after the approximate solution pertaining to buckling behavior is calculated using Matlab software.

2. FORMULATION

A functionally graded porous plate *a*×*b* with parabolic variable thickness (FGP-PVT) is presented in Figure 1. The effective material properties *M*(*z*) through the thickness *h* can be expressed as

$$M(z)=\left[\left(M_c - M_m\right)V_c(z)+M_m\right]\times \\ \times\left[1-e_0\cos\left(\frac{\pi z}{h(x)}\right)\right] \quad (1)$$

with

$$V_c(z)=\left(\frac{z}{h(x)}+\frac{1}{2}\right)^n \\ \text{for} \quad z\in\left[-\frac{h(x)}{2},\frac{h(x)}{2}\right] \quad (2)$$

$$h(x)=h_0\left[1+\left(\frac{x}{a}\right)^2\right] \quad (3)$$

and e_0 is porosity factor, *n* is the power-law index, symbols *m* and *c* represent the metal and ceramic constituents.

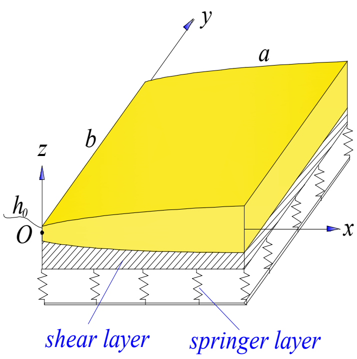

Figure 1. The FGPPVT plate resting on Pasternak foundation

Pasternak's model is determined by

$$\Omega=k_1w(x,y)-k_2\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \quad (4)$$

with k_1 is springer stiffness and k_2 is shear stiffness. Reddy's C^0-TSDT is used to express the plate's displacement field as follows

$$u(x,y,z)=u_0+\left(z-\frac{4z^3}{3h^2(x,y)}\right)\beta_x-\frac{4z^3}{3h^2(x,y)}\phi_x \quad (5)$$

$$v(x,y,z)=v_0+\left(z-\frac{4z^3}{3h^2(x,y)}\right)\beta_y-\frac{4z^3}{3h^2(x,y)}\phi_y \quad (6)$$

$$w(x,y,z)=w_0 \quad (7)$$

Using a four-node quadrilateral element with seven degrees of freedom for each node u_0, v_0, w_0, β_x, β_y, ϕ_x and ϕ_y for finite element procedure similar to the literature [15-21] and finding the buckling load.

3. THE APPROXIMATE SOLUTION

For example, the material properties can be seen in Table 1.

Table 1. The material properties

Al_2O_3	E_c = 380 GPa	ν_c = 0.3	ρ_c = 3800 kg/m^3
Al	E_m = 70 GPa	ν_m = 0.3	ρ_m = 2707 kg/m^3

Firstly, the *a/b* ratio gets values 0.5, 1 & 2 with remaining parameters as h_0=*a*/60, $\bar{k}_1=75$ and $\bar{k}_2=15$. The dimensionless values are presented by

$$\bar{N}_{cr}=\frac{N_{cr}a^2}{E_m h^3},\ \bar{k}_1=\frac{k_1a^4}{D},\ \bar{k}_2=\frac{k_2a^2}{D} \text{ with } D=\frac{E_m h^3}{12\left(1-\nu_m^2\right)}.$$

The buckling loads of the (SCSC) FGP-PVT rectangular plates are shown in Table 2 and compared with other solutions from [14]. It can be seen that the increases in *n* or e_0 reduces the load $\bar{N}_{cr}$. Table 3 further lists the buckling loads of FGP-PVT square plates with input parameters: h_0 =*a*/55, e_0 = 0.2 and *n* = 10.

Table 2. The comparison of buckling load $\bar{N}_{cr}$

a/b	*n*	e_0 = 0: [14]	e_0 = 0: Present	e_0 = 0.2: [14]	e_0 = 0.2: present	e_0 = 0.4: [14]	e_0 = 0.4: present
0.5	0	47.2858	47.3557	43.9859	44.0713	40.6807	40.6667
			0.148%		*0.194%*		*-0.034%*
	2	19.6898	19.9104	18.2596	18.4326	16.7933	16.8321
			1.120%		*0.947%*		*0.231%*
	4	17.3742	17.2634	16.2219	16.1788	15.0427	14.9238
			-0.638%		*-0.266%*		*-0.790%*
1	0	64.8525	65.1179	60.3002	60.8337	55.7386	*55.7142*
			0.409%		*0.885%*		*-0.044%*
	2	26.8777	26.7915	24.9055	24.8712	22.8831	22.6333
			-0.321%		*-0.138%*		*-1.092%*
	4	23.6810	23.7068	22.0922	22.1004	20.4658	20.5885
			0.109%		*0.037%*		*0.599%*
2	0	148.3186	149.4774	137.5344	138.2654	126.7048	127.0001
			0.781%		*0.531%*		*0.233%*
	2	59.7360	60.1206	55.0820	55.1333	50.3075	50.4417
			0.644%		*0.093%*		*0.267%*
	4	52.1433	52.1399	48.3993	48.4189	44.5657	44.3279
			-0.007%		*0.040%*		*-0.534%*

Table 3. The buckling loads $\bar{N}_{cr}$ of FGP-PVT square plates

BCs	$\bar{k}_1$	$\bar{k}_2$	Buckling load
SSSS	0	0	*2.4137*
	100	0	*3.2104*
	100	10	*4.6994*
CCCC	0	0	*6.0783*
	50	0	*6.3528*
	50	5	*7.1067*

4. CONCLUSION

In this article, the buckling load of the FGP-PVT plate is shown. The results of this article are approximate with other solutions in references. The results obtained are anticipated to provide valuable insights for the design of FGP-PVT plates in practical engineering applications.

REFERENCES

[1] A. Pasha and B. M. Rajaprakash (2022), “Fabrication and mechanical properties of functionally graded materials: A review”, *Materials Today: Proceedings*, 52, 379-387.

[2] E. J. Kim, C. M. Lee and D. H. Kim (2023), “Study on the characteristics of functionally graded materials from Ni-20cr to Ti-6Al-4V via directed energy deposition”, *Journal of Alloys and Compounds*, 955, 170263.

[3] D. D. Singh, S. Arjula and A. R. Reddy (2021), “Functionally graded materials manufactured by direct energy deposition: A review”, *Materials Today: Proceedings*, 47, 2450-2456.

[4] Y. Xiong, Z. Han, J. Qin, L. Dong, H. Zhang, Y. Wang, H. Chen and X. Li (2021), “Effects of porosity gradient pattern on mechanical performance of additive manufactured Ti-6Al-4V functionally graded porous structure”, *Materials & Design*, 208, 109911.

[5] D. Khrapov, M. Kozadayeva, A. Koptyug, T. Mishurova, D. Meinel, R. Surmenev and M. Surmeneva (2023), “Geometrical features and mechanical properties of the sheet-based gyroid scaffolds with functionally graded porosity manufactured by electron beam melting”, *Materials Today Communications*, 35, 106410.

[6] H. Chaabani, S. Mesmoudi, L. Boutahar and K. E. Bikri (2023), ”A high-order finite element continuation for buckling analysis of porous FGM plates”, *Engineering Structures*, 279, 115597.

[7] D. Shahsavari, M. Shahsavari, L. Li and B. Karami (2018), “A novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation”, *Aerospace Science and Technology*, 72, 134-149.

[8] L. Hadji, F. Bernard and N. Zouatnia (2022), “Bending and Free Vibration Analysis of Porous-Functionally-Graded (PFG) Beams Resting on Elastic Foundations”, *Fluid Dynamics and Materials Processing*, 19(4), 1043-1054.

[9] F. Z. Zaoui, D. Ouinas and A. Tounsi (2019), “New 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations”, *Composites Part B: Engineering*, 159, 231-247.

[10] M. Taczała, R. Buczkowski and M. Kleiber (2015), “Postbuckling analysis of functionally graded plates on an elastic foundation”, *Composite Structures*, 132, 842-847.

[11] M. Dhuria, N. Grover and K. Goyal (2021), “Influence of porosity distribution on static and buckling responses of porous functionally graded plates”, *Structures*, 34, 1458-1474.

[12] A. Heydari, A. Jalali and A. Nemati (2017), “Buckling analysis of circular functionally graded plate under uniform radial compression including shear deformation with linear and quadratic thickness variation on the Pasternak elastic foundation”, *Applied Mathematical Modelling*, 41, 494-507.

[13] B. Qin, Q. Wang, R. Zhong, X. Zhao and C. Shuai (2020), “A three-dimensional solution for free vibration of FGP-GPLRC cylindrical shells resting on elastic foundations: a comparative and parametric study”, *International Journal of Mechanical Sciences*, 187, 105896.

[14] T.T. Nguyen, T.S. Le, T.T. Tran and Q.H Pham (2024), “Buckling analysis of functionally graded porous variable thickness plates resting on Pasternak foundation using ES-MITC3”, *Latin american journal of solids and structures*, 21(2), e524.

[15] S. Merdaci and H. Belghoul (2019), "High-order shear theory for static analysis of functionally graded plates with porosities", *Comptes Rendus Mécanique*, 347(3), 207-217.

[16] N. Radić (2018), "On buckling of porous double-layered FG nanoplates in the Pasternak elastic foundation based on nonlocal strain gradient elasticity", *Composites Part B: Engineering*, 153, 465-479.

[17] M. Li, C. G. Soares and R. Yan (2021), "Free vibration analysis of FGM plates on Winkler/Pasternak/Kerr foundation by using a simple quasi-3D HSDT", *Composite Structures*, 264, 113643.

[18] A.H. Baferani, A.R. Saidi and H. Ehteshami (2011), "Accurate solution for free vibration analysis of functionally graded thick rectangular plates resting on elastic foundation", *Composite Structures*, 93(7), 1842-1853.

[19] H.L. Ton-That (2020, "Finite element analysis of functionally graded skew plates in thermal environment based on the new third-order shear deformation theory", *Journal of Applied and Computational Mechanics*, 6(4), 1044-1057.

[20] A. Mahmoudi, S. Benyoucef, A. Tounsi, A. Benachour, E.A.A. Bedia and S.R. Mahmoud (2019), "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *Journal of Sandwich Structures & Materials*, 21(6), 1906–1929.

[21] L.H.T Ton (2021), “A modified shear deformation theory associated with the four-node quadrilateral element for bending and free vibration analyses of plates”, *International Journal of Engineering and Applied Physics*, 1(3), 235–241.

[22] T.S Tayeb, M. Amir, S.W Kim and D. Choi (2024), "An analytical modelling of free vibration in porous FG-CNTRC plate resting on elastic foundations", *Materials Today Communications*, 40, p. 109925.

[23] L.X Peng, J.K Sun, Y.P Tao and Z.M Huang (2024), "Bending analysis of thin plates with variable stiffness resting on elastic foundation via a two-network strategy physics-informed neural network method", *Structures*, 68, p. 107051.

[24] M.X Wang, X.B Yan, Y.C Liu and P.H Wen (2024), "Method of fundamental solution for thin plate resting on elastic foundation with in-plane forces", *Engineering Analysis with Boundary Elements*, 164, p. 105767.

[25] H.J Jiang, Q.Z Guo, X.G Wang and N.H Gao (2024), "Comparison of free vibration behaviors for simply supported and clamped T-shaped thin plate resting on Winkler elastic foundation", *Thin-Walled Structures*, 197, 111621.

[26] Z. Dai, H. Tang, S. Wu, M. Habibi, Z. Moradi and H.E Ali (2023), "Nonlinear consecutive dynamic instabilities of thermally shocked composite circular plates on the softening elastic foundation", *Thin-Walled Structures*, 186, p. 110645.

[27] J.R Cho (2022), "Nonlinear bending analysis of FG-CNTRC plate resting on elastic foundation by natural element method", *Engineering Analysis with Boundary Elements*, 141, pp. 65-74.

[28] B.A Selim and Z. Liu (2021), "Impact analysis of functionally-graded graphene nanoplatelets-reinforced composite plates laying on Winkler-Pasternak elastic foundations applying a meshless approach", *Engineering Structures*, 241, p. 112453.

[29] A.H.A. Hassan and N. Kurgan (2020), "Buckling of thin skew isotropic plate resting on Pasternak elastic foundation using extended Kantorovich method", *Heliyon*, 6(6), e04236.

[30] W. Ye, J. Liu, H. Fang and G. Lin (2020), "Numerical solutions for magneto–electro–elastic laminated plates resting on Winkler foundation or elastic half-space", *Computers & Mathematics with Applications*, 79(8), pp. 2388-2410.

[31] D.S Mashat, A.M Zenkour and A.F Radwan (2020), "A quasi-3D higher-order plate theory for bending of FG plates resting on elastic foundations under hygro-thermo-mechanical loads with porosity", *European Journal of Mechanics - A/Solids*, 82, p. 103985.

[32] V.C Haciyev, A.H Sofiyev and N. Kuruoglu (2018), "Free bending vibration analysis of thin bidirectionally exponentially graded orthotropic rectangular plates resting on two-parameter elastic foundations", *Composite Structures*, 184, pp. 372-377.

[33] J. Dong, X. Ma, Y. Zhuge and J.E Mills (2018), "Local buckling of thin plate on tensionless elastic foundations under interactive uniaxial compression and shear", *Theoretical and Applied Mechanics Letters*, 8(2), pp. 75-82.