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CRITERIA AND STANDARDS FOR SELECTING A NEW STATE CARTOGRAPHIC PROJECTION

The selection of a new State cartographic projection is a fundamental cartographic activity at the state level, upon which the quality of spatial data will rely for an extended duration.

The paper shows the basic criteria for the selection of a new State cartographic projection, as well as the evaluation of the proposed projections for the selection of a new State cartographic projection of the Republic of Macedonia.

Keywords: states cartographic projection, Transverse Mercator projection

1. INTRODUCTION

The State cartographic projection is a fundamental cartographic standard, which defines the processing and visualization of spatial data at the national level. All geodetic activities aimed at the acquisition of spatial data, as well as standardized cartographic products, must comply with the standards imposed by the State Cartographic Projection and the State Coordinate System.

When it comes to the Republic of Macedonia, the activities for the selection of a new State cartographic projection began 25 years ago. Those activities are generally the result of the works (scientific topics, doctoral dissertations [3], etc.) of the members of the Chair of Advanced Geodesy at the Faculty of Civil Engineering in Skopje.

The series of papers ([4], [5], [6], [7], [9]), that have been published in the Scientific Journal of Civil Engineering in the past period, which treat the characteristics and the possibility of applying a series of conformal cartographic projections on the territory of the Republic of Macedonia, are in that direction.

Thus, this paper summarizes part of the analyzes and conclusions related to the criteria and standards used in the selection of the new State cartographic projection of the Republic of Macedonia.

2. CRITERIA FOR THE SELECTION OF A NEW STATE CARTOGRAPHIC PROJECTION

The criteria for assessing the quality of projections aim to achieve a comprehensive and optimal selection of the new State cartographic projection.

The analysis of cartographic projections based on the specified criteria should lead to the selection of a national cartographic projection that incorporates recommendations and experiences from international bodies in the field of geodesy and cartography. This selection process should also prioritize the national interests of the Republic of Macedonia in defining the National Spatial Reference System. Some of the mentioned criteria are quantitative, while others are qualitative, introducing additional aspects for evaluating the quality of the projections.

The primary mathematical (quantitative) criteria include:

- Size (maximum and average values) of linear deformations.
- Size of surface deformations.
- Maximum value of meridian convergence.
- On the other hand, qualitative criteria for evaluating projections encompass:
- Regularity in the distribution of linear and surface deformations, expressed through the arrangement of respective isolines.
- Simplicity and completeness of the mathematical model.
- Economic aspects of implementing the new projection.
- Tradition.
- Adherence to contemporary European standards in the field of state cartographic projections.

Research for the selection of a new State cartographic projection begins with the examination of factors influencing the choice and the criteria used to evaluate the proposed projections. As a fundamental factor influencing projection selection, the mathematical-geographical position of the national territory is highlighted, encompassing:

- Spatial distribution,
- Dimensions, and
- Shape of the national territory.

The spatial arrangement of the Republic of Macedonia is defined by its location in the northern hemisphere at around the 42nd parallel and the meridian with a geographical longitude of $\lambda = 22°$. The surface area of the national territory is 25,713 km², with a total length of the border line being 889 km. The maximum distance between two points along the national border is 216.1 km, and the radius of the circle described using the distance between border points and the central point of the national territory is 115.1 km.

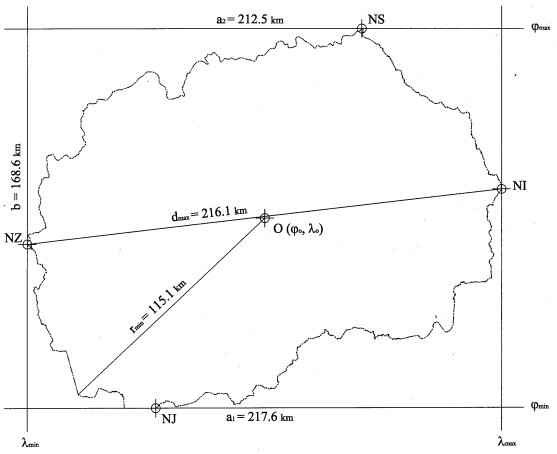

Figure 1. Dimensions and spatial arrangement of the state territory

The territory of the Republic of Macedonia has an ellipsoidal shape, with a gentle elongation in the direction of the parallels.

The spatial arrangement and shape of the national territory indicate suitability for the application of Stereographic and conic projections, while its dimensions allow for the entire territory to be mapped into one coordinate system with satisfactory accuracy.

Research for the selection of the new State cartographic projection begins with analyses of the existing State cartographic projection. The Gauss-Krüger projection was adopted as the State cartographic projection for the Republic of Macedonia in 1924, and its basic properties are well known within the geodetic community [1], [10].

The coordinate system of the projection aligns with the equator, representing the Y-axis, and the meridian with a geographical longitude of $\lambda = 21°$, defining the X-axis of the system. There is constant linear module $m_o = 0.9999$.

Isolines are parallel lines symmetrically arranged concerning the projection of the central meridian.

The results of testing, subject to comparison with the parameters of other projections, point out the shortcomings of the current state cartographic projection. These deficiencies arise from the insistence on mapping the entire national territory into one coordinate system (the seventh), a consequence of the characteristics of the projection and the geographical position of our country. Typically, the part of the territory located east of the meridian 22°30' belongs to the eighth coordinate system.

The insistence on a single coordinate system predicts deformations in the projection ranging from -10 cm/km to 25.5 cm/km, which represents an exceptionally high value.

The second in the series of examined projections is the Tissot projection, serving as the basis for comparing and evaluating the remaining conformal projections in the selection of the new State cartographic projection due to its property of minimizing linear deformations [7].

Although not inherently conformal, the projection is characterized by practical conformality, especially when projecting small territories.

The distribution of linear deformations is regular, and isolines have the shape of concentric ellipses. The maximum accuracy of the projection is achieved by modulating the rectangular coordinates with a linear modulus m = 0.99997, allowing the entire territory of the Republic of Macedonia to be covered with deformations not exceeding ±3 cm/km. This is an absolute minimal value for linear deformations achievable with any of the conformal cartographic projections.

The set of conformal projections, among which the choice for the new State cartographic projection needs to be made, includes:

- Lambert Conformal Conic Projection (Variant V),
- Transverse Mercator Projection,
- Stereographic Projection,
- UTM Projection.

Considering the shape of the national territory, *Lambert's conformal conic projections* stand out as one of the most optimal solutions. This is especially true for Variant V of conformal conic projections, which possesses the property of minimizing linear deformations.

The distribution of linear deformations in this projection is regular, and isolines are represented through projections onto parallels that transform into concentric circles. The coordinate system of the projection is defined by the meridian with a geographical longitude of λo = 21°45', materializing the X-axis, and the tangent to the projection from the parallel with a geographical latitude of φo = 41°36'50", materializing the Y-axis of the coordinate system [5].

The maximum values of linear deformations resulting from the conditions under which the projection is defined are distributed in the range of ±4.4 cm/km.

The *Transverse Mercator projection* shares an identical mathematical foundation and characteristics with the Gauss-Krüger projection.

The central meridian of the projection, materializing the X-axis of the rectangular coordinate system, passes through the center of the national territory, enabling a regular and symmetrical distribution of linear deformations.

Modulating the rectangular coordinates with a constant linear module m=0.99993 allows the entire national territory to be mapped with deformations ranging from -7 cm/km to 7.3 cm/km.

The inclusion of the *Stereographic projection* in the set of examined projections is primarily dictated by the shape of the territory of the Republic of Macedonia. The proper arrangement of linear deformations distributed in the form of concentric circles corresponds entirely to the spatial distribution of the national territory [6].

The center of the Stereographic projection, which simultaneously represents the origin of the rectangular coordinate system, is located at the point with coordinates:

$$\varphi_o = 41^\circ\ 30'\ 30" \quad \lambda_o = 21^\circ\ 45'\ 50"$$

Increased accuracy of the projection is achieved by introducing a secant projection plane, where the rectangular coordinates are modulated with a module m=0.999965. This allows the entire national territory to be covered with deformations not exceeding ±3.8 cm/km.

The last in the series of analyzed projections is the *UTM (Universal Transverse Mercator) projection*. Its significance lies primarily in being an integral part of the World Projection System and a fundamental projection for NATO member countries.

The application of the UTM projection for mapping the territory of the Republic of Macedonia allows adherence to global standards,

particularly in the production of topographic maps [5].

The key characteristics of the UTM projection indicate that it is essentially identical to the Gauss-Krüger projection, differing only in the linear modulus, which here has a value of m=0.9996. This practically means that a negative linear deformation of -40 cm/km is introduced at the central meridian, which is simultaneously the maximum linear deformation for the entire country.

The main characteristics and parameters that appear as criteria for evaluating the projections are presented in Table 1. The analysis of quantitative criteria includes a comparison of characteristics in terms of linear deformations, surface deformations, and meridian convergence:

- The Stereographic projection is characterized by absolute minimal linear deformations. The closest results to it are achieved by the Lambert conformal conic projection, followed by the Transverse Mercator projection, Gauss-Krüger projection, and UTM projection.
- Identical results and order of projections are obtained when analyzing surface deformations.
- The maximum convergence of meridians has the same value for the Transverse Mercator projection and Lambert conformal conic projection. Following them are the Gauss-Krüger projection and UTM projection. The Stereographic projection stands out because it lacks expressions for determining meridian convergence as a function of rectangular and geographical coordinates.

The analysis of qualitative criteria includes other criteria that cannot be mathematically quantified:

- The distribution of linear deformations is regular in all tested projections, except for the Gauss-Krüger projection and the UTM projection, where asymmetry occurs due to the mismatch of the central meridian with the mathematical-geographical positi-on of the Republic of Macedonia.
- The analysis of the mathematical models of the test projections is of interest. Lambert's Conformal Conic projection has the simplest expressions, followed by the Gauss-Krüger projection and the Transverse Mercator projection. The UTM projection, due to its complex surface marking system, lags in this criterion, while the Stereographic projection has the most complex mathematical apparatus.
- The cylindrical conformal cartographic projections, followed by Lambert's conformal conic projection, have the most comprehensive mathematical models. In the Stereographic projection, some expressions for basic calculations are missing.
- The tradition of use in the Republic of Macedonia is on the side of the Gauss-Krüger projection, which has been our national cartographic projection for over 100 years. Due to the identical mathematical models, the Transverse Mercator projection and the UTM projection will follow the Gauss-Krüger projection. The application of Lambert's conformal conic projection in the Republic of Macedonia is

Table 1. Basic characteristics of the test projections [8].

Criteria \ Projection			Gauss-Krüger		Tissot		Lambert conical	Transverse Mercator		Stereographic		UTM
			tangent	secant	tangent	secant		tangent	secant	tangent	secant	
Quantitative	Linear deformations	Max. (cm/km)	+35.4	+25.4	+6.0	±3.0	±4.4	+14.3	+7.3	+7.3	+3.8	-40.0
		Θ_l (test-model) (cm/km)	8.0	8.0	2.3	1.3	2.9	3.7	4.3	2.8	1.7	32.0
		Θ_l (cities) (cm/km)	10.4	8.5	2.4	1.2	2.8	4.4	3.8	3.1	1.5	29.5
	Area deformations	Max. (m^2/ha)	7.1	5.1	1.2	0.6	0.9	2.9	1.5	1.5	0.8	-8.0
		Θ_p (test-model) (m^2/ha)	1.6	1.6	0.5	0.3	0.6	0.7	0.9	0.6	0.3	6.4
		Θ_p (cities) (m^2/ha)	2.1	1.7	0.5	0.2	0.5	0.9	0.8	0.6	0.3	5.9
	Max. convergence		+1° 21' 26.73"		+0° 51' 40"		-0° 51' 26.54"	+0° 51' 29.43"		-0° 51' 54"		+1° 21' 26.7"
Qualitative	Distribution of linear deformations		irregular - asymmetric		regular - ellipses		regular - arcs	regular - lines		regular - circles		irregular - asymmetric
	Mathematical model		relative complex		complex		simple	relative complex		complex		complex - labelling
	Model completeness		complete		incomplete		complete	complete		incomplete		complete
	Tradition in RM		maximum		none		none	maximum		none		moderate
	Economical aspects		none		-		maximum	basic		maximum		increased
	European standards		8% from European countries use as SCP		0% from European countries use as SCP		14% from EU countries use as SCP	61% from European countries use as SCP		8% from European countries use as SCP		8% from EU countries use as SCP

limited to the Civil Aviation Sector. The Stereographic projection does not have a tradition of use in our country.

- According to economic aspects, it is considered that the use of the existing national cartographic projection will not incur additional costs. Considering that it is a projection with a well-known mathematical apparatus, the choice of the Transverse Mercator projection should only incur basic costs necessary for transfor-ming the entire set of spatial data. In contrast, the introduction of the UTM projection, in addition to basic costs, would incur additional (not significant) costs for the education of the professional staff regarding the surface marking system. The most significant additional education costs would be realized if Lambert's conformal conic projection or the Stereographic projection were chosen as the new national cartographic projection.
- Regarding European standards in the field of national cartographic projections, the situation is quite clear. The Transverse Mercator projection has by far the greatest application, followed by Lambert's conformal conic projection, UTM projection, Gauss-Krüger projection, and the Stereographic projection.

Taking into account the analysis conducted, a table can be compiled in which the assessments for the test projections will be systematized in terms of the established criteria. For this purpose, the standard methodology for evaluating the projections was used, in which the weighting of the each criteria was applied for each of the projections (5 points - the best projection, 1 point - the worst projection).

From the results in Table 1, the Transverse Mercator projection clearly stands out as the best choice for a new state cartographic projection for the needs of the national survey and official cartography. While it may have weaker results in some mathematical criteria compared to the Stereographic projection and Lambert Conformal Conic projection, this projection demonstrates excellent performance in qualitative criteria due to its close resemblance to the Gauss-Krüger projection. The widespread use of this projection in most European countries further confirms its favorable characteristics.

The Lambert Conformal Conic projection exhibits excellent properties in terms of mathematical criteria and solid properties in qualitative criteria. However, further investigation is needed to assess its potential use as the official cartographic projection for aerial navigation.

The inadequacy of the Gauss-Krüger projection concerning the mathematical-geographical position of the Republic of Macedonia excludes this projection from the competition for retaining the status of the state cartographic projection. Its primary advantage lies in its extensive tradition of use in our country.

Regarding the UTM projection, it occupies the second-to-last place in our analysis. Its only advantage lies in the alignment of the state cartographic projection with the official cartographic projection for the production of topographic maps according to NATO standards.

Table 2. Evaluation of test projections [8].

Projection / Criteria	Gauss-Krüger	Lambert conical	Transverse Mercator	Stereographic	UTM
Linear deformations	2	4	3	5	1
Surface deformations	2	4	3	5	1
Meridian convergence	3	5	5	1	3
Linear deform. distribution	1	5	5	5	1
Mathematical model	4	5	4	1	2
Completeness of the model	5	2	5	1	5
Tradition in RM	5	2	4	1	3
Economic aspect	5	2	4	1	3
European standards	2	4	5	1	3
Overall	29	33	38	21	22

However, this is not sufficient (considering the series of drawbacks) for the UTM projection to take the place of the new State Cartographic Projection.

In conclusion, despite being characterized by minimal linear and surface deformations, the Stereographic projection, due to the remaining criteria, especially qualitative ones, lags behind the other projections in the set of tested projections for the needs of establishing a new State Cartographic Projection.

3. CONCLUSIONS

Based on all the information presented so far, the conclusion is evident that the **Transverse Mercator projection** represents the best choice for a new State cartographic projection in the Republic of North Macedonia, catering to the needs of national mapping and official cartography.

Of course, it should be noted that the introduction of the new State cartographic projection must be considered in inseparable unity with the introduction of **the new geodetic datum** for the territory of the Republic of Macedonia.

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