ANALYTICAL MODEL OF NON-SWAY STEEL FRAME WITH SEMI-RIGID CONNECTIONS

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The distribution of moments in a steel frame is directly correlated, beside other parameters, by the rigidity of the connection between the constituent elements. Depending on the type of beam-column connection, i.e. depending on its initial rotational stiffness, a diagram of internal static quantities is also generated. In design practice, the traditional way of treating a beam-column connection is carried out by assuming one (or their mutual combination) of the following ideal cases:

1. Absolute rigid connection
2. Absolute pinned connection

This of course, misinterprets the actual behavior of the structure and contributes to unnecessary increased material consumption

The real rigidity of a connection is always between these two extremes cases. Eurocode 3, Part 1-8 provides rules and procedures for quantifying the initial rotational stiffness of a bare steel connection, but does not specify how to determine the moment in the connection joint from the influence of given external loads.

The purpose of this paper is to describe the distribution of moments in a non-sway steel frame with semi-rigid connections under the influence of a uniformly distributed vertical loads and to answer the following question:

Is it more convenient to analyze the beam as an isolated element with semi-rigid connections at the ends or is it necessary to include the rigidity of the vertical elements (columns)?

This goal has been achieved by proposing a two-parameter analytical model that includes both the rigidity of the columns and the rotational rigidity of end beam connections, all together included in one model. The proposed model is compared to the Liu and Chen model [3] which gives the dependence between the external load and the negative moment in the semi-rigid connection, taking into account only the initial rigidity of the beam-column connection.

According to this model for determining the distribution of moments, the rigidity of the vertical construction – columns is neglected. This way of neglecting the columns in analysis

of a steel frame is also adopted in Eurocode 3, Part 1-8.

This of course contributes to the slight overestimation of the negative moment in the beam-column relationship. Determination of the moment in the sagging and hogging region (connection node) from a given external uniformly distributed vertical load is directly dependent on the rotational rigidity of the connection (secant stiffness, S_j) and the rigidity of the vertical structure (k_c).

Keywords: semi-rigid connections, beam-column connection, rotational stiffness, column stiffness, steel frames.

1. PROPOSED TWO-PARAMETER MODEL

The construction of the model starts with the consideration of a bare steel frame with span L_b. The height of floor 1 and floor 2 are $h_1$and h_2 respectively (Figure (1)).

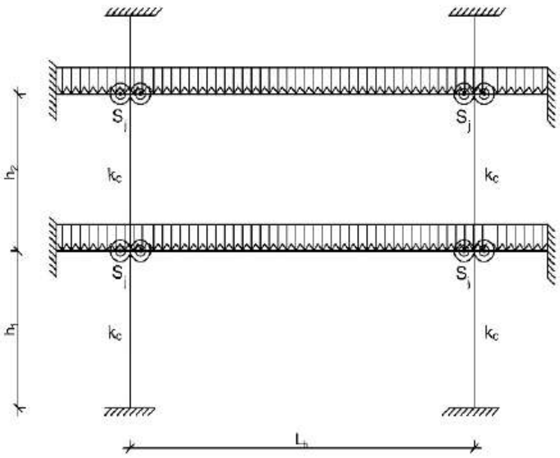

Figure 1. Non-sway bare steel frame with semi-rigid connections

The stiffness of the column beam connection is denoted by $S_{j,ini}$, while the stiffness resulting from the columns is denoted by k_c. To explain the mechanism of transfer of the moment from the beam to the beam-column connection, including column rigidity, an isolated beam is considered as an integral part of the frame, shown in Figure (2).

The correlation between the rigidity of the columns and the rigidity of the beam-column connection is taken as an integral part in the node itself, it is shown in Figure 2 a). The total stiffness that comes from k_c and S_j, is denoted by k and is determined by the equation([4]):

$$\frac{1}{k} = \frac{1}{k_c} + \frac{1}{S_j} \quad (1)$$

Or, equivalently:

$$k = \frac{k_c S_j}{k_c + S_j} \quad (2)$$

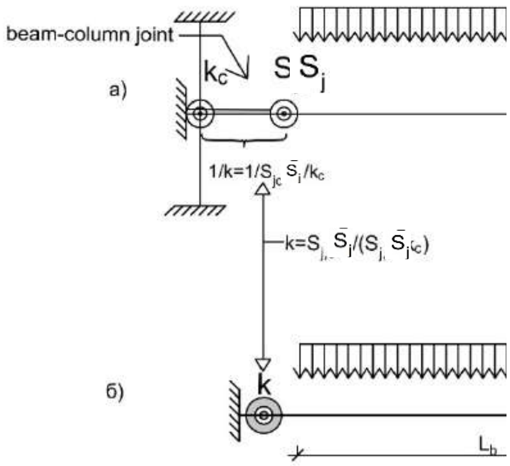

Figure 2. a) Effect of column on beam-column connection b) combining column rigidity k_c and connection stiffness

The parameter k is the parameter "S_j" for the isolated beam, if it is treated according to the philosophy adopted in Eurocode 3, Part 1-8.
In addition, the mechanism of transfer of bending moments from the beam to the connection node is shown, with taking into account the above functional parameters. The bending moment at the connection node is related to the relative rotation between the beam-column. The problem is solved in stages:

1.1 ROTATION - STAGE I

In the first stage, the rotation of a simple supporting beam is determined depending on the intensity of the vertical external load according to the expression:

$$\theta_0 = \frac{ql^3}{24EI_b} \quad (3)$$

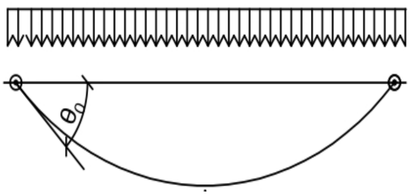

Figure 3. Positive rotation of simply supported beam

1.2 ROTATION - STAGE II

In the second phase, two sub-phases are considered:

- Negative end beam rotation with semi-rigid connection $(S_{j,})$

Depending on the external uniformly distributed vertical load and the unknown amount of bending moment (M_{ed}-which is to be determined by this procedure) that appears in the beam-column connection, the "negative" end beam rotation by including these effects is expressed by:

$$\theta_{sr} = \frac{M_{ed}}{2EI_b} \quad (4)$$

- Negative beam rotation due to the rigidity of the column (k_c)

The relationship between the Negative rotation, the unknown moment in the node (M_{ed}) of the connection and the rigidity of the vertical structure that occurs due to the rigid effects of the column is determined by the following relation:

$$\theta_{kc} = \frac{M_{ed}}{k_c} \quad (5)$$

where k_c is calculated by([5]):

$$k_c = \frac{\alpha_1 E_S I_{c1}}{h_1} + \frac{\alpha_2 E_S I_{c2}}{h_2} \quad (6)$$

- h_1 –story height for level 1,
- h_2 –story height for level 2 и
- the modulus of elasticity of the columns in the corresponding positions. The coefficient α_1 for columns fixed in the base is 4. If the columns are pinned to the base, then the value of the coefficient is α_1 = 3 while the coefficient α_2, would always have the value α_2 = 4.

Together with the above-mentioned, by adopting the principle of superposition of influences, these two sub-mechanisms are combined as shown in the following sketch:

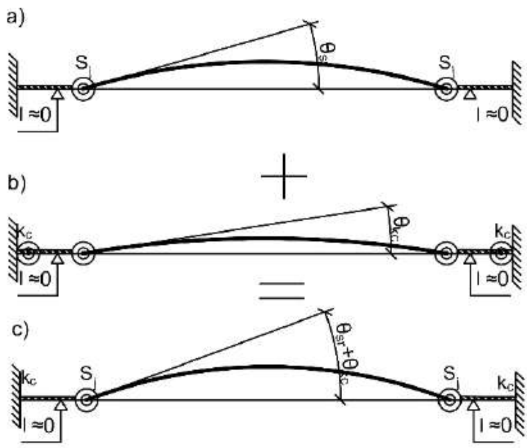

Figure 4. a) Negative rotation due to beam-column stiffness b) Negative rotation due to column rigidity c) Overall negative rotation from last two effects

The sum of the positive rotation of simply supported beam and the last two ,,negative" rotations (based on Figure 4c) is:

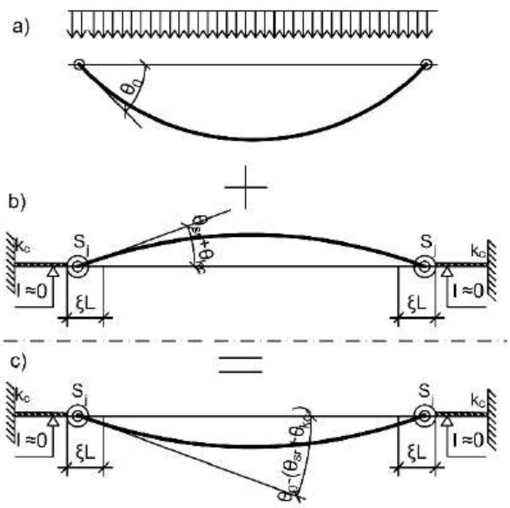

Figure 5. a) Positive rotation due to stage I b) Negative rotation due to stage II c) Overall positive rotation due to combined effects

The last one (Figure 5c), mathematically is represented by:

$$\theta_{S_j} = \theta_0 - (\theta_{sr} + \theta_{kc}) \quad (7)$$

$$= \frac{ql^3}{24EI_b} - \left(\frac{M_{ed}}{2EI_b} + \frac{M_{ed}}{k_c}\right)$$

Where θ_0, θ_{sr} and θ_{kc} denote the rotation of a simple beam, the negative rotation due to the semi-rigid effects at the end beam and the negative rotation due to the rigidity of the columns, respectively.

The correlation between the external load moment (M_{ed}) and the corresponding secant stiffness S_j is given by the well-known expression:

$$\theta_{S_j} = \frac{M_{ed}}{S_j} \tag{8}$$

By equalizing expressions (7) and (8), the hogging moment of the external load at the column-beam connection can be represented as follow:

$$\frac{M_{ed}}{S_j} = \frac{ql^3}{24EI_b} - \left(\frac{M_{ed}}{2EI_b} + \frac{M_{ed}}{k_c}\right) \tag{9}$$

Expressing M_{ed} from (9) gives:

$$M_{ed} = \frac{qL_b^3}{24EI_b} \frac{1}{\left(\frac{1}{S_{j,c}} + \frac{1}{k_c} + \frac{L_b}{2EI_b}\right)} \tag{10}$$

Equation (10) can be transformed into the following form:

$$M_{ed} = \frac{\frac{qL_b^3}{24EI_b}}{\frac{L_b}{EI_b}} \frac{1}{\left(\frac{EI_b}{S_{j,c}L_b} + \frac{EI_b}{k_c L_b} + \frac{1}{2}\right)} \tag{11}$$

If $\frac{S_{j,c}L_b}{EI_b}$ and $\frac{k_c L_b}{EI_b}$ are denoted by R_1 and R_2 respectively, then the last expression would take the final form:

$$M_{ed} = \frac{2R_1R_2}{3(2R_1 + 2R_2 + R_1R_2)} \cdot M_{sag}^0 \tag{12}$$

The proposed two parameter model (12) incorporates the parameters R_1 and R_2 and the sagging bending moment M_{sag}^0 if the beam is considered as a simply supported beam. That is, the moment of the external load in the connection node is given as a functional dependence on the following parameters:

$$M_{ed} = M_{ed}(M_{sag}, S_j, k_c) \tag{13}$$

The coefficient $\frac{2R_1R_2}{3(2R_1+2R_2+R_1R_2)}$ is always a number less than 1. In other words, this quotient is a reduction of the sagging moments of a simply supported beam and the same reduction is transmitted from region of positive moment to the node of the semi-rigid beam-column connection.

2. DESCRIPTION OF ONE-PARAMETER MODEL ACORDING TO LIU AND CHEN

As a comparison, if we do not take into account the participation of the vertical construction then a change in the law of moments instead of a two-parameter function (12) would be described using a graph of a function of one main variable (S_j)). That's only with one parameter i.e. the parameter R_1. The next paragraph describes the philosophy of this approach.

The total end beam rotation is assumed to be the sum of the rotation when the beam is considered to be simply supported by an uniformly distributed load and the "negative" rotation when the beam is exposed to a negative moment due to the semi-rigid effect that causes the beam to "overhang".

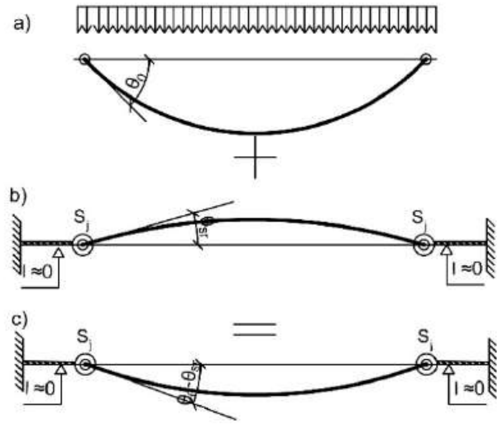

Figure 6. a) Positive rotation due to stage I b) Negative rotation due to beam-column rotational stiffness c) Overall positive rotation due to combined effects

Where:

- θ_0: Rotation of a simply supporting beam by the action of an uniformly distributed vertical load
- θ_{sr}: Negative rotation due to the effect of a semi-rigid connections
- S_j: Secant rigidity of bare steel semi-rigid connection

In this model, it is evident that the columns do not participate in the distribution of moments.
As in previous model, the superposition principle is assumed so the total rotation of the end beam node can be defined as:

$$\theta_j = \theta_0 - \theta_{sr} \tag{14}$$

$$\theta_0 = \frac{qL_b^3}{24EI_b} \tag{15}$$

$$\theta_{sr} = \frac{ML_b}{2EI_b} \tag{16}$$

By substituting equations (16) and (15) in equation (14) we get:

$$\theta_j = \frac{qL_b^3}{24EI_b} - \frac{M_{ed}L_b}{2EI_b} \tag{17}$$

By equalizing equations (17) and (8), the moment in the semi-rigid connection node is determined according to the relation:

$$M_{ed} = \frac{2R_1}{3(R_1+2)} M_{sag}^0 \tag{18}$$

Where:

- R_1: coefficient of rotational stiffness which is $R_1 = \frac{S_{j,c}L_b}{EI_b}$.
- M_{sag}^0: The moment in sagging region when the beam is treated as simply supported.

3. COMPRASION OF THE PROPOSED MODEL WITH THE ONE PARAMETRIC MODEL

To have a better understanding on differences between the proposed model and the standard one parametric model [3], the following example is considered:

A bare steel frame with two stories under the influence of uniformly distributed vertical load of $g_{ed} = 12.5\ \mathrm{kNm}$ is considered. The columns are selected as HEB160 with height of $h_1 = 4\mathrm{m}$ and $h_2 = 3\mathrm{m}$. The beam is selected as IPE240 with span $L_b = 8\mathrm{m}$.

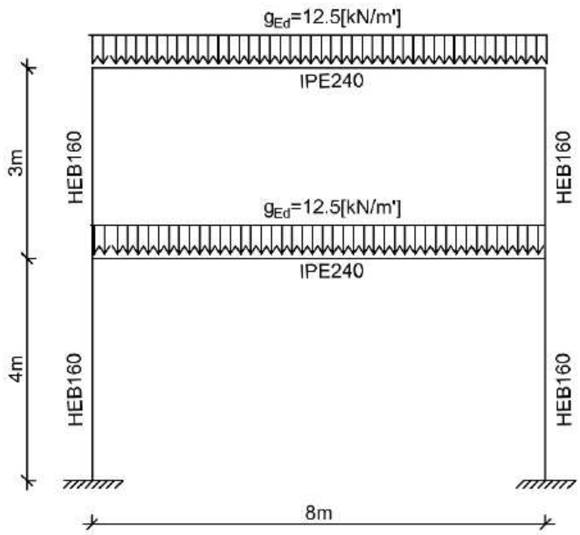

Figure 7. a) Positive rotation due to stage I b) Negative rotation due to beam-column rotational stiffness c) Overall positive rotation due to combined effects

The initial rotational stiffness and the secant stiffness of the beam-column connection calculated in [5] are: $8816\ \mathrm{kNm/rad}$ and $4408\ \mathrm{kNm/rad}$, respectively. With the implementation of the proposed method, the moment in the connection node (M_{hog}) as well as sagging moment (M_{sag}) for the first story are determined by calculating all the necessary parameters. The results for the proposed model are shown in the table below.

Table 1. Calculation according to the proposed model

Parameter	**Calculated parameters**
$\mathbf{S_{j,ini}}$	$\mathbf{8816\ kNm/rad}$
$\mathbf{S_j}$	$\mathbf{4408\ kNm/rad}$
$\mathbf{R_1}$	$\mathbf{4.31}$
$\mathbf{k_c}$	$\mathbf{12210.8\ kNm}$
$\mathbf{R_2}$	$\mathbf{11.92}$
$\mathbf{\frac{2R_1R_2}{3(2R_1+2R_2+R_1R_2)}}$	$\mathbf{0.4}$
$\mathbf{M_{sag}^0}$	$\mathbf{100\ kNm}$
$\mathbf{M_{hog}}$	$\mathbf{40\ kNm}$
$\mathbf{M_{sag}}$	$\mathbf{60\ kNm}$

From the shown table calculations, it is noticed that the positive bending moment, M_{sag}, is $60\ \mathrm{kNm}$

In the following table, the calculation of the positive moments, M_{sag}, for the same example was carried out, but with the implementation of the one-parameter model.

Table 2. Calculation according to the one-parameter model

Parameter	Calculation of parameters
$\mathbf{S_{j,ini}}$	$\mathbf{8816\ kNm/rad}$
$\mathbf{S_j}$	$\mathbf{4408\ kNm/rad}$
$\mathbf{R_1}$	$\mathbf{4.31}$
$\mathbf{\frac{2R_1}{3(R_1+2)}}$	$\mathbf{0.46}$
$\mathbf{M_{sag}^0}$	$\mathbf{100\ kNm}$
$\mathbf{M_{hog}}$	$\mathbf{46\ kNm}$
$\mathbf{M_{sag}}$	$\mathbf{54kNm}$

From the performed analysis, it is noticed that, for the same frame and the same level of loads, the positive moment for the first story is $54\ \mathrm{kNm}$.

A comparison of the obtained results is described in the following sketch:

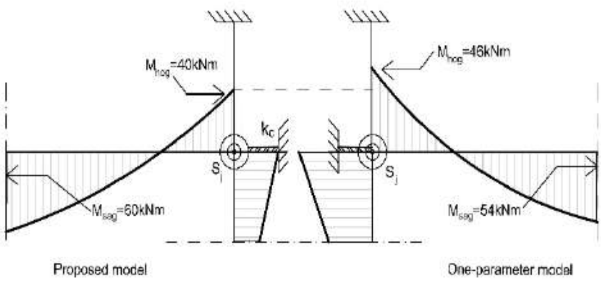

Figure 8. Hogging and sagging moments calculated by proposed model and their comparison with moments calculated by one-parameter model.

4.CONCLUSION

This paper presents a two-parameter analytical model for determining the negative moment in a semi-rigid bare steel, beam to column connection.

In this model, in addition to the secant rigidity of the beam-column connection, the rigidity of the vertical construction is considered through the parameter $\mathrm{k_c}$. The purpose behind the introduction of this parameter was to assess whether the neglection of the column rigidity has a favorable or unfavorable effect on the distribution of moments in a bare steel frame.

Next, a one parameter model based on Liu and Chen([3]) is presented and their mechanism of transferring the bending moment in a bare steel frame is compared.

The reason for this comparison (with a one-parameter model) is precisely due to the fact that the philosophy of dealing with this issue in Eurocode 3 is based on this one-parameter model.

From the conducted analysis, with the two-parameter model, for the same frame and the same level of loads, the sagging moment is approximately 10% higher compared to the one-parameter model.

As a final conclusion can be drawn that, analysing a beam as an isolated beam with semi-rigid connections and excluding column rigidity implies underestimating sagging moment and overestimating the hogging moments.

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