Stefan Stankovski
MSc
University “Ss. Cyril and Methodius”
Faculty of Civil Engineering – Skopje
stefan@prostor.mk
stefan.stankovski.94@hotmail.com

Sergey Churilov
PhD, Associate Professor
University “Ss. Cyril and Methodius”
Faculty of Civil Engineering – Skopje
Blvd. Partizanskiodredi 24, 1000 Skopje
curilov@gf.ukim.edu.mk

DYNAMIC ANALYSIS OF BELL TOWER WITH BELL FORCES

Bell towers are structures dating a few thousand years ago while their structural behavior is discovered in detail only now in the last few decades. They are monuments of culture and civilization, witnessing the time when they were built but also witnessing the history of the region around them. Old bell towers are built as a masonry structures and as that stand the test of time to this day. The new ones are made of reinforced concrete, steel or combination of both materials.

This research paper presents rational procedure for determination of dynamic characteristics of bell tower in order to have an insight into its true structural behavior and to check occurring of resonance during the ringing of the bells. This methodology was conducted on the bell tower of Cathedral Church „St. Clement of Ohrid” in Skopje.

Keywords: bell tower, swinging bell forces, church bells, resonance, natural frequencies

1. INTRODUCTION

It is a known fact that first bells originated in the region of today Armenia and were widely used by the great ancient civilizations like Egypt, China, Greece and Rome. Bells had a great role in the people’s everyday life because they were used like instrument for signalization that it is time to pray or for holy rituals like weddings, funerals etc. In Christianity, bells are used since 4th century A.D. and from Italy they were successfully spread to Spain, France and Britain where they started to signal the time too. Over the years, cities grew larger and so did the bells. Hence the need to place the bells on a tower that will be high enough so that the bells can be heard as far away as possible and at the same time to accommodate the bells. This tower is called bell tower and first appeared around 11th century. At first it was built as masonry structure and only from the 20th century bell towers are built as reinforced concrete or steel structures. Around the 15th century bells finally got the characteristic form that is known today [1].

Big bell towers usually have big bells with weight over 1000 kg and are placed on the upper levels which present big concentrated masses. When bells begin to ring, they swing around their axis of rotation and with that they generate horizontal and vertical dynamic forces which are first transmitted to the bearing sub structure and later to the tower itself.

Towers are cantilever like structures and thus concentrated masses at the end subjected to dynamic loading will cause large horizontal displacements. When it comes to dynamic actions, the displacements can be amplified by the effect of resonance. Basically, if one of the natural frequencies of the tower is close enough to the frequencies of the swinging system than the induced forces by the bells motion will be amplified.

Generally, it is known that the maximum vertical force can reach values up to four times the weight of the bell and the maximum horizontal force up to 2 times the weight of the bell [2]. Because of the structural system of the bell tower, it possesses bigger axial stiffness than lateral or torsional stiffness. Therefore, the vertical forces from the swinging bells are not a problem for the structural behavior and are not taken into account. As for the horizontal forces, they present big challenge for the bell tower and can cause big horizontal displacements which can cause structural damage the bell tower or lead to collapse.

Accurate determination of swinging bell forces is an important step when making an overall structural analysis of a bell tower. These forces are generated due to the mass of the bells or in other words due to their inertia of the ringing system and only through dynamic approach these forces can be accurately obtained.

In the past engineers used empirical formulas to assess the horizontal force induced by the swinging bell. Bell towers older than a century are surely designed with these empirical formulas and nowadays, throughout the world, these towers cannot perform normally because the masonry material had deteriorated over time and the tower can no longer accept dynamic loading from the swinging bells.

Bell towers are usually slender structures and may be highly sensitive to the dynamic actions induced by wind, earthquakes and motion of bells. The dynamic forces caused by the motion of the bells are one of the main actions that influence the structural behavior of bell towers every day. Also, the frequencies of these dynamic forces have important interaction with the natural frequencies of the supporting structure. The knowledge of the dynamic action induced by the movement of the bells is of great importance in the structural safety assessment as well as in the restoration of bell towers[3].

Traditionally it has been considered sufficient to keep odd harmonic oscillations of the ringing bell off the fundamental natural frequencies of the tower. The German standard for bell towers, DIN 4178-2005, explicitly suggest that the difference between these two frequencies should be greater than 20% in order to avoid resonance[4].

2. CHARACTERISTICS ABOUT CONSIDERED BELL TOWER

This paper deals with the bell tower of Cathedral church of “St. Clement of Ohrid” in Skopje which is a few decades old. It is fully separated from the church, as shown on Figure 1and Figure 2, it is located in the center of the city and has coordinates of 41.998853, 21.426139. The tower is made out of poured reinforced concrete except for the façade panels which are prefabricated.

The structural system can be seen on Figure 3 and is contained of: beams, slabs, shear walls and cantilever slabs at the top levels. It can be noticed that the structural system has no columns which means that all vertical loading is accepted by the four shear walls. Also, the bell tower has a ground floor, Figure 4, plus ten floors.

Figure 1. Position of the bell tower

Figure 2. View of the bell tower

Figure 3. Structural system

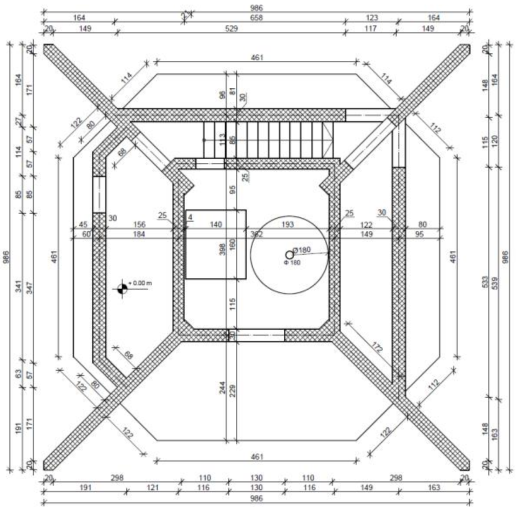

Figure 4. Ground floor layout

Relevant project documentations for the bell tower were not found and therefore it was manually measured by measuring tape and laser. Later, it is planned to conduct measurement with LiDAR method, a geodetic method for measurement that produce high quality point cloud model of the measured object in 3D where all unreachable distances can be measured. Also, with this method the manually measured distances will be additionally confirmed.

This bell tower has three bells located on the top floor and their disposition can be seen on Figure 5. The disposition of the bells has a small plane eccentricity and can cause torsion in the global behavior of the bell tower. These bells are casted in Innsbruck, Austria. In some bell towers where the bells are accommodated at the windows the plane eccentricity is large enough and can cause significant torsion which is not favorable since bell towers have low torsional stiffness. Bell 2 has biggest mass of all other bells and the other two are with smaller masses.

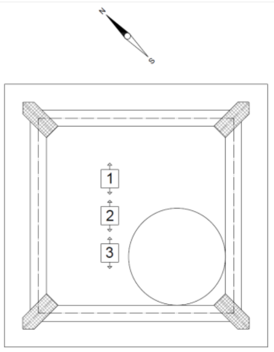

Figure 5. Bells disposition

3. MECHANISM OF ROTATING BELLS

Churches can have one or more bells. The bells can be stationary, rung by a mechanical hammer, with swinging clapper or they can rotate around axis. This depends by the size of the bell itself.

The mechanism of rotating bells consists of an electric motor, a chain and a wheel. The electric motor generates torque which is then transmitted on the wheel through the chain. The axis of the bell is fixed on the wheel which by spinning rotates the bell. This system is shown on Figure 6

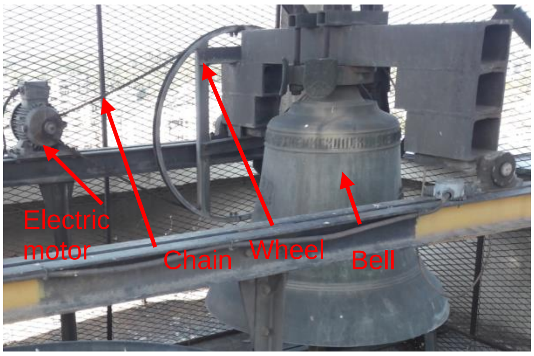

Figure 6. Rotating system

As far as dynamic characteristics are considered, the most important characteristics for the ringing bell are:

- Hits per second
- Max. angle of rotation
- Frequency of the rotating system

- Mass inertial moment(with respect to the axis of rotation)

These characteristics directly affect the dynamic behavior of the ringing bell. If it is needed to change some of these characteristics, one can only adjust the electric motor since it will affect the whole ringing system.

3.1 BELL'S PARTS

The geometry of the bell has been changing throughout the years. Around the 15th century, the bell gained its definitive shape that is known today. For better understanding and description of the bell, its parts are named as show on Figure 7(*1.bell yoke 2.headstock 3.canons 4.shoulder 5.waist 6.soundbow 7.lip 8.mouth 9.clapper 10.bead line*) [5].

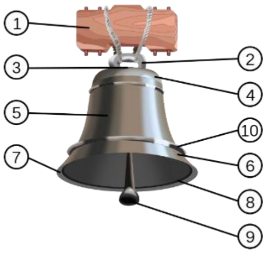

Figure 7. Bell's parts

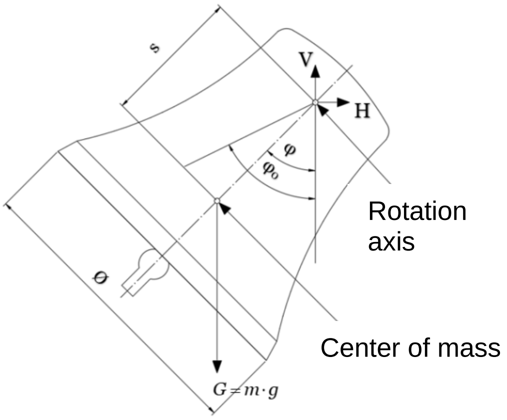

Figure 8. Additional parameters

3.2 RINGING BELL CHARACTERISTICS

During the swinging of the bell, it is out of balance and several additional parameters must be observed as shown on Figure 8.

- φ – angle between vertical and bell axis
- φ_0 – Maximum angle the bell can reach
- s – distance between axis of rotation and the center of mass
- V – vertical force generated from the swinging bell
- H – Horizontal force generated from the swinging bell

While the bell is swinging over time the angle φ changes its values from positive to negative maximum angle. The φ angle directly affect the analytical calculation of the forces H and V and in the meantime this angle depends only on the maximum angle φ_0. While the bell is swinging the two forces H and V are keeping it in balance while the value of the parameter s shows how well the bell is balanced. Good balanced bells have low value for s which leads to lower mass inertial moment of the bell which finally leads to lower forces generated by the swinging bells.

3.3 BELL'S MATERIAL

Throughout the years, bell's material has been changing due to its imperfections. After centuries of experience, innovations and technological advances the bells finally took the shape that is known today. This is due to the fact that a precise mixture of metals for casting church bells has been discovered.

Since the beginning, bells are casted with copper or special alloy of copper, known as bronze. This alloy is precise mixture of 78% copper and 22% tin. Even though the base materials are soft their mixture is pretty solid, little bit elastic, compact, less ductile and resistant to oxidation. The patina layer which forms on the surface of the bell due to oxidation, protects the bell from further oxidation and has green color.

The most important quality of a church bellis to maintain resonance when it is struck and to vibrate and produce the holy om sound. The clapper is usually made of the same material as the bell itself and it can hit the bell with speeds up to 1000 km/h. The specific weight of the bronze is 8900 kg/m^3. Knowing this it is easy to determinate the weight of the bell needed for the further calculations.

3.4 RINGING SYSTEMS

Nowadays there are three main systems of ringing bells. Local people developed these systems and embedded their culture in these structures.

These systems are:

- Central European system

(Central Europe, USA, Canada, Italy, Eastern Europe and some countries in South America)

- English system

  (Great Brittan, USA, Canada, Australia, New Zeeland, South Africa)

- Spanish system

  (Spain, south France, USA and some countries in South America)

The systems have certain differences between them like: direction of ringing, the maximum angle that the bell can reach and the location of the bell itself. As for the location of the bell, in the Central European system Figure 9 and in the English system Figure 12 the bells are located inside the bell tower at top levels. The bells are supported on separate substructure, traditionally made out of massive wood to dampen the vibrations from the bells, nowadays usually made out of steel, while in the Spanish system Figure 10Figure 11 the bells are hung on a wall or a window of the church.

Figure 9 Bells accommodated inside bell tower (C. European system)

Figure 10 Traditional Spanish bell tower (Spanish system)

Figure 11 Modern bell tower (Spanish system)

Figure 12. English system

The angle between the vertical and bell's axis is constantly changing over time while the bell is rotating. In C. European systems this angle can reach values 30° ÷80°while the bell swings alternately. In the English system the bells make full circle while changing the direction every cycle. The bell begins the cycle turned upside down, makes full circle 360° and returns to the starting position upside down. These two systems are very unbalanced because of the bigger distance from the center of mass of the bell and the axis of rotation. In the Spanish system the bells rotate continuously in one direction with a wooden counter weight placed on top of the bell which balances the system. This means that less force will be needed to rotate the system as opposed to the C. European and English system. Such bell on a bell tower generates smaller horizontal and vertical forces [6], [7].

In Macedonia and the nearby region, church's bells are rung according to C. European system. The bell tower that is considered in this paper has bells that are rotating according to C. European tower.

3.5 BELL 2 CHARACTERISTICS

In the considered bell tower only bell 2 is in function and its main characteristics are given in Table 1. According to the conducted measurements the geometry parameters of bell 2 are obtained and with these parameters a 3D model of bell 2 is made in AutoCAD shown in Figure 13 and Figure 14. The bell is made out of bronze.

Figure 13. 3D model of bell 2

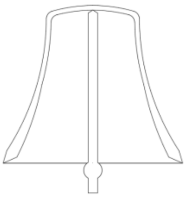

Figure 14. Cross section of bell 2

From the 3D model of the bell it is easy to determinate its center of mass and the position of the rotating axis. From these two parameters the value of "s" is obtained. The value of mass inertial moment is calculated in Eq. (1).

$$I = m \cdot s^2 = 978 \cdot 0.3772^2 = \\ = 139.16\ \mathrm{kg} \cdot \mathrm{m}^2 \qquad (1)$$

The oscillation frequency range of normal bells is in the range of 0.3 to 0.6 Hz. Since the third harmonic oscillation of the bell is predominant in C. European systems, the critical natural frequency of the bell tower is from 0.9 to 1.8 Hz. In slender towers often is the case of the first natural frequencies to be in this range [8].

Table 1. Bell 2 characteristics

Bell 2	Symbol	Value	Unit
Material	$\gamma_{(bronze)}$	8900	kg/m^3
Volume	V*	0.109	m^3
Mass	m	978	kg
Distance between center of mass and axis of rotation	s	0.3772	m
Mass inertial moment	I	139.16	kg·m^2
Maximum angle	φ_0	29	°
Hits per minute	A	55	1/min
Diameter	Ø	1.13	m
Height	h	1	m
Thickness	t	0.03	m

4. CALCULATION OF BELL FORCES

There are several research papers that deal with the bell's forces. The work of Bennati et al.(2005) is of particular importance in terms of calculation of bell forces. The authors make complex experimental measurements to obtain real forces over time and then form mathematical model of a pendulum that yields results almost identical to the experimental ones.

Real motion of a bell is complex, forced and damped at the same time. This movement, mathematically, can be described by a moving pendulum with identical geometric and dynamic characteristics Figure 15. Such a system approximates the mass of the bell at one point, the center of mass, which is connected with rigid rod at a certain distance from the axis of rotation. This approximation gives sufficiently accurate results for the calculation of bell forces.

On the one hand, when the bell moves at the axis of rotation two reactive forces appear H and V, which keep the system in balance Figure 16. While on the other hand the movement of the bell's mass generates active forces in tangential direction of the movement and in the direction normal to the movement Figure 17. Putting all these forces in balance, while changing the angle φ gives the answer to the pendulum i.e. the change of the horizontal and vertical forces [9].

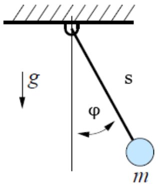

Figure 15. Mathematical model of the pendulum

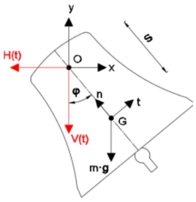

Figure 16. Reactive bell forces

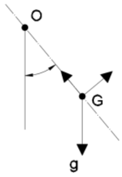

Figure 17. Active bell forces

The dynamic of the bell with the clapper is extremely complex and difficult to analyze due to the nonlinear characteristics, repetitive strokes and complicated excitation. Because of that it is of interest to have a relatively simple model that will produce accurate results.

There are two ways to describe the movement of a bell. The first case is when the bell and the clapper move separately and the second case is when they move together i.e. they have the same angular velocity. In the first case the system has two degrees of freedom while in the second case the system has one degree of freedom. Because the bell has much bigger mass than the clapper case two is adopted.

The impact of the clapper in the bell causes high frequency vibrations that are transmitted to the bearing substructure. Older wooden load bearing structures have filtered out these high frequencies while today's metal structures do not filter these vibrations but instead transfer them to the bell tower.

Considering the bell as a rigid body with a mass m rotating around an axis of rotation O, the horizontal H(t) and the vertical V(t) component of the dynamic loads caused by the ringing of the bell can be represented by Eq. (2) and Eq. (3)

$$H(t) = m \cdot s \cdot [\dot{\varphi}^2 \cdot \sin\varphi(t) - \ddot{\varphi}^2 \cdot \cos\varphi(t)] \qquad (2)$$

$$V(t) = -m \cdot g - m \cdot s \cdot \begin{bmatrix} \dot{\varphi}(t)^2 \cdot \cos\varphi(t) + \\ \ddot{\varphi}(t) \cdot \sin\varphi(t) \end{bmatrix} \qquad (3)$$

The pendulum with one degree of freedom is one of the most studied and analyzed nonlinear systems. The periodic motion of the pendulum is harmonious only when it comes to small oscillation angles. Once this threshold is exceeded, the equations for motion become nonlinear. Usually this threshold is around 10°. The motion of bell 2 occupies an angle much larger than 10° and therefore trigonometric functions are off the table since this problem is of nonlinear nature. The dynamic equilibrium of a bell pendulum is governed by nonlinear, second-order differential equation Eq. (4).

$$I \cdot \ddot{\varphi}(t) + m \cdot g \cdot s \cdot \sin\varphi(t) = 0 \qquad (4)$$

Starting with these few equations the aim is to find a way to calculate the value of φ over time and later its first and second derivatives. With further expansion of Eq. (4) the equation for calculating φ(t) is obtained Eq(5) where sn is Jacobian elliptic function and k is its module. The module k has a certain value from 0 to 1. If k=0 a sine function is obtained for φ(t) and if k=1 a singularity is obtained for the same function. Comparation of sin and sn functions are shown onFigure 18.

$$\varphi(t) = 2 \cdot \arcsin\left[k \cdot \mathrm{sn}\left(\sqrt{\frac{g}{l_r}} \cdot t\right)\right] \qquad (5)$$

$$k = \sin\frac{\varphi_0}{2} \qquad (6)$$

sn – Jacobian elliptic function

k – module of Jacobian elliptic function

g – gravity constant

l_r – reduced length of equivalent pendulum ($l_r = I/(m \cdot s)$)

From Eq. (5) and Eq. (6) it can be noticed that the motion of the pendulum depends only on the maximum angle and the reduced length of the pendulum.

Considering the horizontal forces H(t), the results obtained from the analysis are shown onFigure 19. Calculation of the vertical forces is not taken into account since the tower has great axial stiffness.

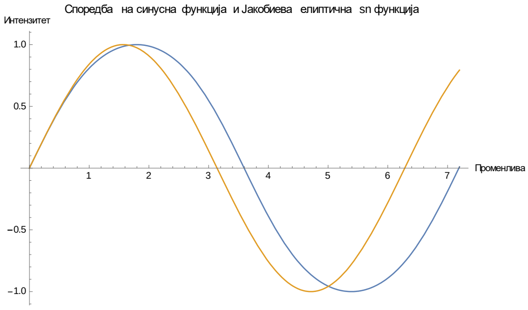

Figure 18. Comparison of sin (orange) and sn (blue) functions

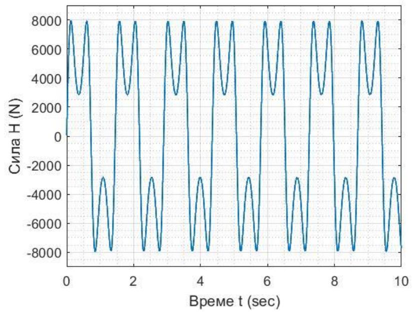

Figure 19. Horizontal force H(t)

From today's literature,[7], it is known that for C. European system of rotation the frequency of third harmonic oscillation from the horizontal force is predominant f_H. Due to this fact, care should be taken to ensure that this frequency is far enough from the natural frequencies f_0 of the bell tower so that resonance does not occur.

Table 2 Frequencies

	1st	2nd	3rd	4th	5th
f_0	**1.6**	1.68	3.98	4.31	5.27
f_H	0.68	**1.37**	2.06	2.75	3.44

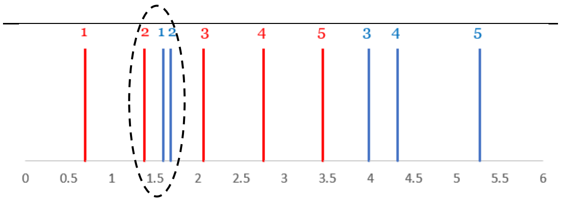

Figure 20. Frequencies comparation (red-harmonic oscillations, blue-natural frequencies)

Table 2and Figure 20 show that low level of resonance is present in the bell tower of Cathedral church "St. Clement of Ohrid" because the difference between first natural frequency and the second harmonic oscillation of the horizontal force is around 15%. The German standard for bell towers DIN 4178-2005 explicitly states that to eliminate resonance the smallest difference between the natural frequencies of bell tower and frequencies of harmonic oscillations should be at least 20%. When resonance occurs, it is expected during the ringing of the bells the horizontal displacements to be amplified and hence cause greater structural damage.

There are several methods that modify the dynamic characteristics of the bell or the bell tower that eliminate the phenomenon of resonance. One of them is to change completely the system of the ringing bells. Another method is to modify the electric motor that drives the rotation of the bell. Also, there is a method where a steel rods are added to the bell tower to stiffen it in order to move away the problematic natural frequency from the harmonic oscillations of the bells [10].

5. CONCLUSION

Considering the stability of a bell tower it can often be severely undermined by the dynamic actions generated by the motion of its bells. In such situations, accurate reconstruction of the dynamic actions transmitted on the tower becomes essential. The procedure presented above for bell forces calculation can be used to accurately predict the motion of bells and their reactions on bell towers.

The aim of this research was to check whether resonance occurs when the bells are ringing. The value of the maximum obtained horizontal force is 82% of the bell's weight. As far as the values of these forces are considered the obtained horizontal forces do not present threat to the structural system. However, if the resonance effects are considered, a low level of resonance is present in the tower. Even though in most bells ringing in the C. European system the frequency of the third harmonic oscillation of the horizontal force presents threat for resonance to occur, in this case the second harmonic oscillation causes the resonance. This frequency pairs with the first natural frequency of the bell tower.

REFERENCES

[1] S. Bennati, L. Nardini, and W. Salvatore, 'Dynamic behavior of a medieval masonry bell tower. Part I: Experimental measurements and modeling of bell's dynamic actions', *J. Struct. Eng.*, vol. 131, no. 11, pp. 1647–1655, 2005.

[2] J. Heyman and B. D. Threlfall, 'Inertia forces due to bell-ringing', *Int. J. Mech. Sci.*, vol. 18, no. 4, pp. 161–163, 1976.

[3] L. Vincenzi, E. Bassoli, F. Ponsi, C. Castagnetti, and F. Mancini, 'Dynamic monitoring and evaluation of bell ringing effects for the structural assessment of a masonry bell tower', *J. Civ. Struct. Heal. Monit.*, vol. 9, no. 4, pp. 439–458, 2019.

[4] DIN-4178-2005, 'DIN (Deutsches Institut fur Normung) (2005) DIN 4178 Glockturme Berechnung und Ausfhrung. DIN, Berlin, in German', no. April. DIN, Germany, p. 32.

[5] 'Church bell - Wikipedia'. [Online]. Available: https://en.wikipedia.org/wiki/Church_bell. [Accessed: 21-Jun-2020].

[6] S. Ivorra and J. R. Cervera, 'Analysis of the dynamic actions when bells are swinging on the bell-tower of Bonrepos i Mirambell Church (Valencia, Spain)', *Proc. 2002 Int. Conf. Noise Vib. Eng. ISMA*, pp. 2343–2348, 2002.

[7] S. Ivorra, F. J. Pallarés, and J. M. Adam, 'Masonry bell towers: Dynamic considerations', *Proc. Inst. Civ. Eng. Struct. Build.*, vol. 164, no. 1, pp. 3–12, 2011.

[8] H. et al. Bachmann, *Vibration Problems in Structures.pdf*. BASEL: BIRKHAUSER VERLAG, 1995.

[9] S. Stankovski, 'Modeling, structural identification and dynamic analysis of bell tower of Cathedral church "St. Clement of Ohrid" - Skopje', 'Ss. Cyril and Methodius' University - Skopje, Macedonia, 2020.

[10] M. Lepidi, V. Gattulli, and D. Foti, 'Swinging-bell resonances and their cancellation identified by dynamical testing in a modern bell tower Swinging-bell resonances and their cancellation identified by dynamical testing in a modern bell tower', no. October 2017, 2009.