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NUMERICAL PROCEDURE FOR STABILITY CALCULATION IN INELASTIC DOMAIN

The problems of instability of steel frame structures have been of interest to many investigations for a long time. The fast development of computer technology has created the possibility of a complete solution to such a problem. This paper briefly presents one numerical procedure for the global stability analysis that is based on the FEM. The stiffness matrices are derived using the trigonometric shape functions. Also, when the buckling of the structure occurs in the inelastic domain, the tangent modulus theory is applied. Obtained results show justification for applying such a procedure for the stability calculation of the frame structures.

Keywords: inelastic buckling, tangent modulus, finite element method

1. INTRODUCTION

The study of the stability of linear structures begins with the first Euler's researches in the 18th century [7]. His and many other solutions were mainly based on determining the differential equation of buckling according to the second order theory. Later, for the analysis of more complex structures, the researchers had to make some simplifications [14]. So, the compressed elements of the frame were extracted from the whole structure. Corresponding boundary conditions introduce the presence of other structural beams and girders that are connected with the analyzed one. In this case, the results of the stability analysis are presented by the approximate formulas and diagrams, separately for the braced and unbraced frames [1]. These solutions then served to formulate procedures for multi-story frame calculation. The methods most commonly used for such analysis were slope deflection method and stiffness distribution method [1,12]. Such a theory of isolated member later becomes the basis of many recommendations and regulations for the stability analysis of the frame structures, for example [9,8]. The design of such structures is defined by the determination of the effective length factor k. However, despite the frequent use of such an approach, it has numerous limitations, as it was shown in [2]. First of all, such calculation does not consider the rigidity of the whole structure. Also, the influence of inelastic material behavior and imperfections are not taken into account [3]. The development of the finite element method enabled the global stability analysis of frame structures [11]. In its usual procedure, it reduced to solving a well-established eigenvalue problem. This paper will outline the procedure where the shape functions are used in the trigonometric form, according to the exact solution of stability differential equation. Also, material nonlinear behavior will be considered using the tangent modulus concept [10].

2. APPLICATION OF THE TANGENT MODULUS THEORY

The main goal of this analysis is to formulate, conditionally speaking, the exact procedure for the stability calculation of steel frames. Therefore, as it is already emphasized, it is applied finite element method with interpolation functions in the trigonometric or hyperbolic form. So, the buckling problem will be reduced to the solution of the transcendental equation which depends, in a very complicated way, upon the axial forces in columns and beams [3].

Corresponding stiffness matrices also have to take into account nonlinear material behavior. The tangent modulus concept has been used for that purpose. The concept of the tangent modulus was introduced in [6], a later developed by many other authors, for example [10,13]. So, in the “classic” elastic analysis Young's modulus of elasticity (E) should be replaced with tangent modulus (E_t), which is stress dependent function. This module is used to estimate the gradual yielding effect due to residual stresses along the length of members under axial loads [2]. The expression that is used in this analysis was firstly suggested in [5], and latter accepted in some relevant researches [13]:

$$\mathrm{E}_t = 4 \cdot E \cdot \left[\frac{\sigma}{\sigma_y} \cdot \left(1 - \frac{\sigma}{\sigma_y} \right) \right] \tag{1}$$

where σ_y is yield stress.This is empirical an expression designed to represent the perfor-mance of structural steel columns in the inelstic domain [10]. This formula is valid for $\sigma > 0.5 \cdot \sigma_y$.

To solve this buckling problem it is important to formulate corresponding stiffness matrices. The procedure for deriving these matrices is presented in [4]. Here is given only the stiffness matrix of the member that is is clamped at both ends and subjected to compression axial force, Eq.(5).

$$\omega_t = \sqrt{\frac{P_{cr,i}}{E_t \cdot I}} \cdot l \tag{2}$$

$$\Delta_t = 2 \cdot (1 - \cos\omega_t) - \omega_t \cdot \sin\omega_t \tag{3}$$

where with is $P_{cr,i}$ is marked critical force in the observed member.

When the global stiffness matrix is defined, it is possible to calculate the critical load from the homogeneous matrix equation:

$$\boldsymbol{K}\boldsymbol{q} = 0 \tag{4}$$

where **q** is the vector of generalized coordinates. This problem can be solved by an incremental process, i.e. finding the solution for det(**K**) = 0.

The real challenge in this research was to carry out an appropriate numerical analysis. Namely, as it was already emphasized, the buckling problem is reduced to the solution of the transcendental equation which depends, in a very complicated way, upon the normal forces in columns and beams [3]. Although in this case only one finite element is needed for each column or beam, the problem is more complicated to solve than the “usual” eigenvalue problem for which there are several established methods. So, for this purpose, an appropriate computer program was developed using the C++ programming language and it is presented in detail in [4]. The basic possibilities of this program are analysis according to the first and the second-order theory and stability analysis of linear frames. This paper briefly presents only part of an algorithm related to the calculation of the critical load in an elastic and inelastic domain.

The program firstly needs to calculate the critical load in the elastic domain. The first iteration is performed according to the first-order theory. Then obtained forces are used as initial values in the second-order theory calculation. Afterward, the program continues the iterative calculation until the displacement difference in two consecutive iterations

$$\boldsymbol{K} = \frac{E_t I}{l^3 \Delta_t} \begin{bmatrix} \omega_t^3 \sin\omega_t & \omega_t^2 l (1-\cos\omega_t) & -\omega_t^3 \sin\omega_t & \omega_t^2 l\,(1-\cos\omega_t) \\ & \omega_t\, l^2 (\sin\omega_t - \omega_t \cos\omega_t) & -\omega_t^2 l (1-\cos\omega_t) & \omega_t\, l^2 (\omega_t - \sin\omega_t) \\ & & \omega_t^3 \sin\omega_t & -\omega_t^2 l (1-\cos\omega_t) \\ & simetr. & & \omega_t\, l^2 (\sin\omega_t - \omega_t \cos\omega_t) \end{bmatrix} \tag{5}$$

becomes smaller than some pre-set small value. At the end of this procedure, the stiffness matrix of the whole system is obtained, and only the active degrees of freedom are considered. That stiffness matrix must satisfy the condition for the existence of the nontrivial solution. The final result of this procedure gives the critical force for the constant modulus of elasticity (i.e. in the elastic domain).

After that, columns where the critical stress ($\sigma_{cr}=P_{cr}/A$) greater than the proportionality limit (σ_p) need to change their stiffness. Namely, tangent modulus (E_t), according to Eq.(1), should be introduced for them. Columns with σ_{cr} which did not reach σ_p should keep "old characteristics". Then, the iterative calculation should be performed again in the same way as for the "elastic" analysis. As a result, this procedure gives the corresponding critical load factor and the value of tangent modulus for all elements buckling in the inelastic domain.

3. NUMERICAL EXAMPLES

Only some of the calculation possibilities of this program will be presented in this paper. In all examples is used steel with characteristic: E = 210,000,000 kN/m^2 and σ_y = 240,000 kN/m^2. First results are obtained by variation of columns stiffness for the simplest one-story sway frame (Figure 1).

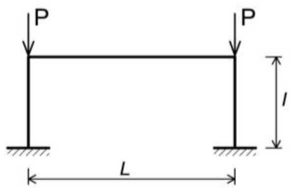

Figure 1. One-story sway frame

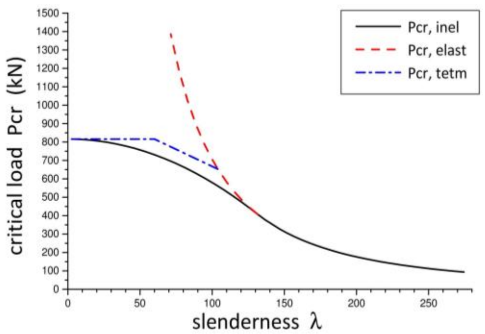

Figure 2. Pcr – λ diagram for the frame (Fig.1)

The critical column stress variation vs. slenderness ratio for the one-story sway frame is presented in Figure 2. The results of the presented elastoplastic analysis are shown by a solid line. Euler curve (results of elastic stability analysis) is marked with a dashed line. The dash-dotted line represents Tetmajer's linear formula [13]. From these results, obtained by a self-developed program, it can be seen the usefulness of applying the analysis in the inelastic domain, particularly for the columns with low slenderness (or short columns).

The critical stress–strain curve for analyzed numerical example is given in Figure 3. As it was expected, below a proportionality limit, modulus of elasticity E is constant, and above this point, a nonlinear relationship between stress and strain is obtained.

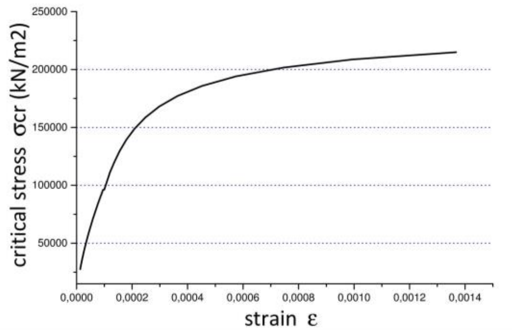

Figure 3. σ_{cr} – ε curve for the frame (Fig.1)

It is clear that these results represent a good verification of the proposed program. Similar results for the two-story and six-story frames are given in [4].

In order to illustrate some other possibilities of the presented numerical procedure, a steel five-story three-bay sway is considered, Fig.4.

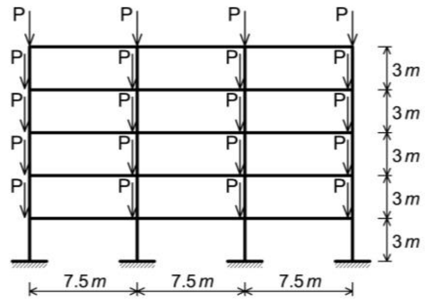

Figure 4. Five-story three-bay sway frame

Rigid connections for columns ends including their supports at the base are assumed. Concentrated loads P are imposed on each column at each story level. Since the axial load in the columns increases from the stories above to the base level, presented elastoplastic stability analysis leads to the different behavior of the columns in the different levels. Six different IPB cross-sections of the columns are assumed for the parametric analysis, while floor beams are IPE300.

Table 1 gives the critical load values for the analyzed cross-sections for elastic and proposed elastoplastic stability analysis. It is clear that the application of proposed stability analysis in the inelastic domain is justified for such a numerical example. Namely, this is a rather rigid frame and it can be exposed to significant axial load. In the case of similar frames [3] but with higher columns, this difference in result is smaller.

Table 1. P_{cr} for the frame (Fig.4)

	$P_{cr,el}$	$P_{cr,inel}$
IPB120	364.3	146.0
IPB160	851.9	245.2
IPB200	1383.2	360.6
IPB240	1875.5	495.3
IPB280	2315.5	616.9
IPB320	2825.9	751.1

The corresponding values of the tangent moduli (for only two floors) at the moment of buckling are presented in Table 2.

Table 2. E_t for the frame (Fig.4)

	tangent moduli	
	3rd floor	1st floor
IPB120	208,861,665	79,146,292
IPB160	206,514,563	46,863,965
IPB200	204,992,189	30,691,635
IPB240	204,063,803	21,716,697
IPB280	203,404,726	15,644,936
IPB320	202,790,302	12,550,149

From these results it can be observed the difference in the behavior of the upper and lower columns. Overall buckling of this type of structure is governed by the behavior of the columns that are subjected to the highest axial load. Those are the columns on the first floor. The columns in the upper are loaded with smaller axial forces, so their characteristics (ie. modulus of elasticity) have not changed too much.

It should be noted that the above procedure can give some other results of stability problems. For example, the determination of the effective buckling length and the calculation of the load-bearing capacity of the compressed members are presented in [3].

4. CONCLUSIONS

The goal of this paper was to indicate some of the possibilities of a program that was developed according to the presented theoretical procedure. This procedure is based on the global stability analysis of steel frame structures. The calculation was performed for the elastoplastic stability analysis when the geometrically non-linear process is followed by the development of the material nonlinearity as well. Stiffness matrices were derived using the tangent modulus that is a stress-dependent function. Analyzed numerical examples confirm that this procedure is a good alternative for the stability calculation of steel frames in the inelastic domain.

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