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CRACKING IN FLEXURAL HIGH-STRENGTH CONCRETE ELEMENTS SUBJECTED TO VARIABLE LOAD

In this paper the influence of loading histories, including variable (imposed) actions, on the behavior of high-strength reinforced concrete beams were analyzed especially the crack parameters: crack width and crack distance. For the evaluation of long-term effects (effects due to creep and shrinkage in concrete structures), quasi-permanent combination of actions was used to verify the serviceability reversible limit state. Extensive experimental program was performed in order to define the factor ψ_2 for the evaluation of serviceability reversible limit state for crack control with two specific loading histories. Loading histories are consisted of sustained permanent action *G* and repeated variable load *Q* applied in cycles loading and unloading for 24 and 48 hours respectively for the beams series D and E. A total of four reinforced concrete beams, dimensions 15/28/300cm were tested. The beams were made of concrete class C60/75. Experimental results obtained during testing of the beams, from measured maximum crack spacing and crack width, were analyzed by the crack control models given in EN1992-1-1 Eurocode 2 and in the fib Model Code for Concrete Structures 2010.

Keywords: high-strength concrete, crack width, crack spacing, variable load, ψ_2 quasi permanent coefficient

1. INTRODUCTION

Cracks can be usually observed on the concrete surface during service life of concrete structures and causes nonlinear behavior of concrete structures exceeding the tensile strength of concrete. Beside their great influence on serviceability, cracks are also associated to durability, permeability and aesthetics issues.

There are various types of cracks, essentially defined by the principal cause or mechanism, but a few of them can be controlled by the designer. Usually restrained deformations from shrinkage or temperature movements and loading can be treated by the designer [1].

Variable actions such as imposed loads for buildings are those arising from occupancy. Because of nature of variable loads, they have phenomenon of appearance in different time intervals that cannot be predicted and that are acting like random variables during the service life of structure [2]. Repeated variable actions cause significant increase in concrete and reinforcement strain, increase in crack width and deflections, reduction of tension stiffening and increase in bond-slip [3].

The quasi-permanent combination of actions is normally used for long term effects and the appearance of the structure, and usually can be expressed as [4]:

$$\sum_{j\geq 1} G_{k,j} + P + \sum_{i>1} \psi_{2,i} Q_{k,i} \tag{1}$$

Where: $G_{k,j}$-permanent actions; P-prestressing action; $Q_{k,i}$-accompanying variable actions and ψ_2-quasi permanent factor depending on the type of action.

2. MODELS FOR CALCULATION OF CRACK WIDTH

2.1 MODEL OF EUROCODE 2

For the calculation of crack width in reinforced concrete elements following expression may be used [5]:

$$w_k = S_{r,max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right) \tag{2}$$

Where: $S_{r,max}$-maximum crack spacing; ε_{sm}-mean strain in the reinforcement under the relevant combinations of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening and ε_{cm}-mean concrete strain between cracks.

The difference of the main strains in the reinforcement and concrete ε_{sm}-ε_{cm} may be calculated from the expression [5]:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq \geq 0.6\frac{\sigma_s}{E_s} \tag{3}$$

Where: σ_s -stress in the tension reinforcement assuming a cracked section; αe -ratio of E_s/E_c; $\rho_{p,eff}$ -$A_s/A_{c,eff}$ and k_t -factor dependent on the duration of load (k_t =0.6 for short term loading and k_t =0.4 for long term loading).

For the calculation of maximum crack spacing the following expression may be used [5]:

$$s_{r,max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff} \tag{4}$$

Where: ϕ-bar diameter, c-cover to the longitudinal reinforcement; k_1-coefficient which takes account the bond properties of the bonded reinforcement (k_1=0.8 for high bond bars and k_1=1.6 for bars with an effectively plain surface), k_2-coefficient which takes account of the distribution of strains (k_2=0.5 for bending and k_2=1.0 for pure tension), k_3 =3.4 and k_4 =0.425.

2.2 FIB MODEL CODE FOR CONCRETE STRUCTURES 2010

For all stages of cracking, the design crack width w_d may be calculated as [6]:

$$w_d = 2l_{s,max}\left(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}\right) \tag{5}$$

Where: $l_{s,max}$-is the length over which slip among concrete and steel occurs; ε_{sm}-average steel strain over the length $l_{s,max}$; ε_{cm}-average concrete strain over the length $l_{s,max}$; and ε_{cs}-strain of concrete due to shrinkage.

For the length $l_{s,max}$ the following expression applies [6]:

$$l_{s,max} = k \cdot c + \frac{1}{4} \cdot \frac{f_{ctm}}{\tau_{bms}} \cdot \frac{\varphi_s}{\rho_{s,ef}} \tag{6}$$

Where k-empirical parameter to take the effect of the concrete cover into consideration (k=1); c-concrete cover; and τ_{bm}-mean bond strength between steel and concrete.

The relative mean strain ε_{sm}-ε_{cm}-ε_{sh} is [6]:

$$\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = \frac{\sigma_s - \beta \cdot \sigma_{sr}}{E_s} - \eta_r \cdot \varepsilon_{sh} \tag{7}$$

Where: σ_s-steel stress in a crack; σ_{sr}-maximum steel stress in a crack in the crack formation stage, which for pure tension is:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{s,ef}}\left(1 + \alpha_e \rho_{s,ef}\right) \tag{8}$$

$$\rho_{s,ef} = \frac{A_s}{A_{c,ef}} \tag{9}$$

Where: $A_{c,ef}$-effective area of concrete in tension; α_e-modular ratio E_s/E_c; β-an empirical coefficient to assess the mean strain over $l_{s,max}$

depending on the type of loading; η_r-coefficient for considering the shrinkage contribution; and ε_{sh}-shrinkage strain.

The value for τ_{bm} and coefficients β and η_r are given in fib Model Code 2010 for Concrete Structures [6].

3. EXPERIMENTAL PROGRAM

3.1 DESCRIPTION

An experimental program was proposed to analyze long-term behavior of reinforced concrete elements under the action of different types of loading histories. In this paper experimental results from testing of 6 beams were given for the series of beams *D*, *E* and *F*. This part of the experimental program is given in Table 1.

Table 1. Experimental program

	Type of load	Loading cycle
D	Permanent load "G" and variable load "Q"	Loading/unloading for Δ_{t1}=24 hours.
E	Permanent load "G" and variable load "Q"	Loading/unloading for Δ_{t2}=48 hours.
F	Shrinkage	/

Series of beams *D* and *E* were consisting of combination of action of long-term permanent load with intensity *G* and repeated variable load *Q* which was applied in cycles of loading/unloading for 24/48 hours respectively for a period of 330 days.

Beams from series F were used for measuring free shrinkage of reinforced concrete in same period of 330 days.

Design characteristics of actions are given in Table 2. The self-weight of the beam is uniformly distributed load of 1kN/m.

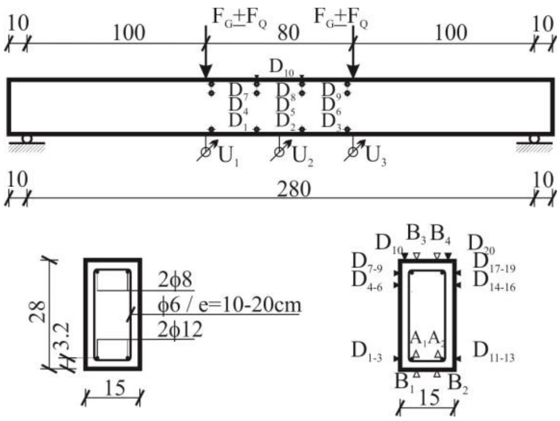
Table 2. Design values of actions

Actions		Intensity [kN]
Permanent action	"G"	2x4
Variable action	"Q"	2x7.6
Service load	"G+Q"	2x11.6

In each series of reinforced concrete beams, the dimensions were width/height/length = 15/28/300cm. Series of beams *D* were divided in *D1* and *D2* made from ordinary concrete class C30/37 and *D3* and *D4* were made of high-strength concrete class C60/75.

This was also applied and for series *E* and *F*.

Details of the beams and test set up for the experiment are provided on Figure 1. All specimens were cast from the same batch of concrete and all specimens were tested at concrete age of 40 days and at concrete age of 370 days.



*D-mechanical deform-meter, A-strain gauge for reinforcement, B-strain gauge for concrete, U-deflection-meter

Figure 1. Reinforced concrete beams, dimensions, detail of reinforcement and test set up

The measured compressive strength, tensile splitting strength and elastic modulus of concrete class C60/75 at the age of loading at 40 days were: f_{ck}=66.4MPa, $f_{ct,sp}$=5.3MPa and E_{cm}=39470MPa. Measured values of concrete properties at age of 370-day for concrete C60/75 were f_{ck}=75.5MPa, $f_{ct,sp}$=5.3MPa and E_{cm}=41230MPa, total shrinkage (as a sum of autogenous and drying shrinkage) ε_c=683x10^{-6} and creep coefficient φ_c=0.703.

Deformed reinforcement, diameter of 12mm, was used with yield strength of $f_{0.2}$=400MPa and modulus of elasticity E_{sm}=200200MPa.

Throughout the period of 330 days the beams were carefully monitored in the middle of the span to record: deflections *a*, development of cracks, number of cracks, l_{smax}-maximum crack spacing, wk–characteristic crack width and for σ_s -steel stress in a crack.

The tests were performed at the Laboratory of Faculty of Civil Engineering, University “Ss. Cyril and Methodius” in Skopje. The environmental conditions in the laboratory were relative constant value of humidity RH=63% and temperature T=17^{o}C.

More details of the experimental program, mix design and results are given in the doctoral thesis of Arangjelovski [2] and in the papers from Arangjelovski, Markovski & Mark [7] and [8].

3.2 RESULTS FROM MEASURED CRACK PARAMETERS

At the start of the experiment at concrete age of *t*=40days, first the beams were loaded by the permanent load *G* which does not caused cracks in the section, then the variable load *Q* was applied and the load *G+Q* causes cracks in the beams. First the crack width w_{G+Q} *(t*=40) was measured approximately in the middle of the span, and then after unloading at the level of permanent load *G* crack width w_G (*t*=40) was measured.

The values of initial crack width, obtained at loading at age of concrete of *t*=40 days and final crack width measured at concrete age of *t*=370 days for series *D* are given in Table 3, and for series *E* in Table 4.

Table 3. Experimentally measured crack width *w* for series "D" beams

Level of actions	Crack width *w*	
	D3- C60/75	D4- C60/75
	[mm]	[mm]
w_G (t_0=40)	0.050	0.040
w_G (t=370)	0.055	0.045
w_{G+Q} (t_0=40)	0.100	0.070
w_{G+Q} (t=370)	0.120	0.080

Table 4. Experimentally measured crack width w for series "E" beams

Level of actions	Crack width *w*	
	E3- C60/75	E4- C60/75
	mm	mm
w_G (t_0=40)	0.06	0.05
w_G (t=370)	0.09	0.08
w_{G+Q} (t_0=40)	0.08	0.07
w_{G+Q} (t=370)	0.12	0.12

One representative diagram of relation crack width *w* versus time *t* was given in Figure 2, for the beam D3 made of high-strength concrete C60/75.

The same was done and for the beam E3 made of high-strength concrete C60/75, a typical diagram of relation between crack width *w* and time *t* is given in Figure 3.

Because of the type of loading histories (repeated loading and unloading) the diagram of the measured crack *w* during time *t* has a form of an area defined by the limits of permanent load G and by the sum of the permanent load G and variable load Q.

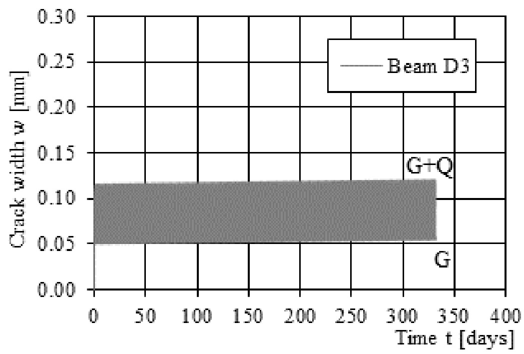


Figure 2. Diagram crack width *w* - time *t* for beam D3 concrete class C60/75

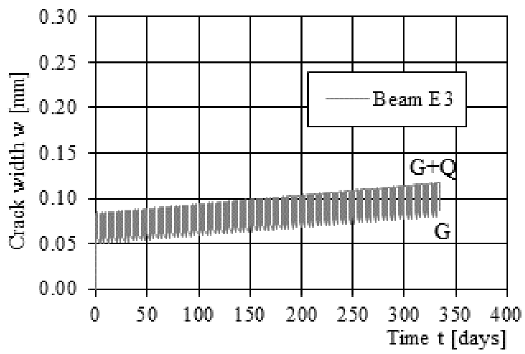


Figure 3. Diagram crack width *w* - time *t* for beam E3 concrete class C60/75

Experimental results from measuring the *n*-number of cracks and $S_{r,max}$ maximum crack spacing is given in Table 5 and Table 6 for series of beams D and E respectively.

Table 5. Experimentally measured maximum crack spacing $S_{r,max}$ for series "D" beams

Beams	No. of crack	Maximum crack spacing $S_{r,max}$	
	n	D3- C60/75	D4- C60/75
		[mm]	[mm]
D3	3	194	238
D4	3	182	180
		182	178
Mean value:		186	199

Table 6. Experimentally measured maximum crack spacing $S_{r,max}$ for series "E" beams

Beams	No. of crack	Maximum crack spacing $S_{r,max}$	
	n	D3- C60/75	D4- C60/75
		[mm]	[mm]
E3	3	202	192
E4	3	200	186
		182	194
Mean value:		195	191

For the purpose of using the model for the calculation of the crack width according to fib Model Code for Concrete Structures 2010, also the free shrinkage was investigated in the experimental program on separate series of beams F made from high-strength concrete class C60/75.

The experimental results for the shrinkage deformation ε_{cs} during the period of t=370 days are given in Table 7.

Table 7. Experimentally measured shrinkage ε_{cs} for series "F" beams

Days	Shrinkage ε_{cs}	
	F3- C60/75	F4- C60/75
	[10^{-3}]	[10^{-3}]
t_0=40	0.070	0.080
t=370	0.160	0.116

4. NYMERICAL ANALYSIS

To compare the experimental results with the results obtained from analytical analysis, evaluation of the serviceability limit state was performed using the crack control model of the EN1992-1-1 Eurocode 2 Design of concrete structures – Part 1-1: General rules and rules for buildings 2004 and fib Model Code for Concrete Structures 2010.

In both models, for the serviceability limit states design, combination of actions was used to verify the serviceability reversible limit state including time effects from shrinkage and creep of concrete, using the quasi-permanent combination of action to verify time-dependent final crack width at the level of permanent load G, which is of interest to define the quasi-permanent coefficient ψ_2.

For the calculation of stresses in the cross section the Age-Adjusted Effective Modulus method was used [9].

4.1 ANALYSIS OF RESULTS FOR MAXIMUM CRACK SPACING $S_{R,MAX}$

First, analysis of maximum crack spacing $S_{r,max}$ was performed to verify the experimental results and analytical results using both crack models. The results of the comparison are given in Table 8.

Using the crack model given in Eurocode 2, calculated maximum crack spacing in the beams made from concrete class C60/75 is similar to the obtained experimental results in range of 5% difference.

Table 8. Experimental and analytical results for maximum crack spacing $S_{r,max}$

	Maximum crack spacing $S_{r,max}$	
Beams	D3- C60/75	D4- C60/75
	mm	mm
Mean value:	186	199
	E3- C60/75	E4- C60/75
Mean value:	195	191
EN1992-1-1	190	190
fib 2010	207.6	207.6

Using the crack model given in fib Model Code for Concrete Structures 2010 overestimated the results for the beams made from concrete C60/75 in range of 10.4% difference.

4.2 ANALYSIS OF RESULTS FOR MAXIMUM CRACK WIDTH W

Analysis of comparison the experimental results and calculated crack width w using both models are given in Table 9.

Table 9. Experimental and analytical results for crack width w

Level of actions	Crack width w	
	D3- C60/75	D4- C60/75
Experiment	mm	mm
w_{G+Q} (t_0=40)	0.10	0.07
w_{G+Q} (t=370)	0.12	0.08
	E3- C60/75	E4- C60/75
w_{G+Q} (t_0=40)	0.08	0.07
w_{G+Q} (t=370)	0.12	0.12
EN1992-1-1 EC2		
w_{G+Q} (t_0=40)	0.11	0.11
w_{G+Q} (t=370)	0.13	0.13
fib Model Code 2010		
w_{G+Q} (t_0=40)	/	/
w_{G+Q} (t=370)	/	/

The crack model given in Eurocode 2, to verify serviceability irreversible limit state, gives proper prediction of the crack width w for the beams made of concrete C60/75. Calculation of the crack width was performed for the level of load as a sum of permanent load G and variable load Q at the time of loading t=40 days and for t=370 days.

In the analysis using the fib Model Code 2010 crack model the crack width could not be calculated for the beams using concrete class C60/75 because the calculated steel stress in the crack σ_s was lower than σ_{sr} maximum steel stress at the crack formation stage $\sigma_s<\sigma_{sr}$.

One explanation for this problem may be that higher mechanical properties of high-strength concrete enables formation of cracks at the level of combination of actions as a sum of permanent *G* and variable load *Q*. The service load, flexure moment M=12.6kNm, is much too close to cracking moment M_{cr}=11.6kNm, which suggest that crack formation stage will last during the whole period of loading.

4.3 ANALYSIS OF RESULTS FOR THE QUASI-PERMANENT COEFFICIENT Ψ2

For the reversible serviceability limit state, quasi-permanent combination of actions was used to verify the crack width at the level of permanent load G, which is from prime interest for the designers. Eurocode 2 crack model was used to obtain the same crack width with experimental ones. The results from the analysis are given in the Table 10.

Table 10. ψ2 factors for series of beams D and E made of high-strength concrete C60/75

No.	G	Q	ψ_2	G+ψ_2Q	Crack width *w*
	kN	kN		M [kNm]	mm
Beams made from concrete C60/75					
D	4	7.6	0.55	9.2	0.05
E	4	7.6	0.70	10.3	0.08

5. CONCLUSIONS

From the experimental and analytical analysis of crack parameters, for beams subjected to permanent load G and repeated variable load Q following conclusions can be received:

-Using both crack models, given in the Eurocode 2 and in fib Model Code 2010, give good agreement with experimental results.

-It is very important, especially when we use high-strength concrete for the reinforced concrete elements to ensure that the tensile steel stress in the crack σ_s are greater than maximum steel stress in the crack in the crack formation stage σ_{sr}. This condition was not satisfied using the fib Model Code 2010.

-For the beams made of concrete C60/75, verification of crack width using quasi-permanent combination of actions shows that the quasi-permanent factor is in range of ψ_2=0.55-0.70. These values are lower than the proposed values in Eurocode 2, which indicates that because of higher mechanical properties we should use higher level of load intensity.

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