

PRACTICAL APPLICATION OF RELIABILITY-BASED DESIGN WITH EXAMPLES INCLUDING RELIABILITY ASSESSMENT OF DESIGN APPROACH DA2*

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In this paper we look at the application of reliability-based design to various types of problems in a geotechnical design office. For the selected design examples, comparisons are presented between the results from reliability analyses and those from traditional design methods such as working stress design and limit states design using design approach DA1-2. In addition, we compare the reliability of eccentrically loaded footings designed according to DA2 and DA2*. Our conclusion is that reliability-based design can readily be applied to problems with closed form solutions provided sufficient data is available to adequately characterise the input parameters. Eccentrically loaded footings designed according to DA2* are less reliable than those designed using design approach DA2 from EN1997-1. Limit states design using design approach DA1-2 from EN1997-1 achieves fairly uniform levels of reliability for different types of structures and is suitable for routine design.

Key words: Reliability-based design; Geotechnical design; Limit states design; Design approaches.

1. INTRODUCTION

A number of factors have combined to facilitate the application of reliability-based design methods in geotechnical engineering practice. These include faster computers, more efficient solution algorithms and increased understanding of the likely variation of input parameters to geotechnical design problems. In this paper, we compare the reliability of common geotechnical structures designed in accordance with Design Approach 1 Combination 2 (DA1-2) from EN1997-1. In the case of eccentrically loaded spread footings, this comparison is extended to include design approach DA2 from EN1997-1 and the proposed DA2* design approach for spread footings.

2. METHODS OF RELIABILITY ANALYSIS

The First Order Reliability Method (FORM) and Monte Carlo simulations can readily be applied to problems with closed-form solutions, i.e. problems where the solution can be found by substitution of the design parameters into equations which have explicit solutions. Examples of such problems are the bearing capacity of a footing or the stability of a retaining wall. In the context of a design office, FORM analyses may be undertaken using spread-sheets such as that developed by Low & Tang (2007). This spreadsheet requires a performance function g to be defined such that failure occurs when $g < 0$. Using the terminology of the Eurocodes, the performance function can be expressed as the difference between the effect of actions (E) and resistance (R), i.e. $g = R - E$. The FORM algorithm determines the value of the reliability index β as the distance in units of standard deviation from the mean point (point in multi-dimensional parameter space where all parameters assume their mean value) to the closest point on the failure surface boundary ($g = 0$), known as the design point. The probability of failure is computed by approximating the failure domain as a linear plane tangential to the failure surface at the design point.

3. EXAMPLES

The examples chosen were based on Orr et al (2005) and are shown in the first column of Figure 1. All these problems have closed form solutions. In each example, the design determines one controlling dimension of the structure, such as the width of a footing or the length of a pile. A single soil type was assumed throughout, a cohesionless sand with a friction angle $\phi = 32^\circ$ and a density $\gamma = 20 \text{ kN/m}^3$. The statistical distributions, coefficients of variation and ratios of characteristic to mean loading were based on Retief and Dunaiski (2010) and Phoon and Kulhawy (1999). The characteristic values for imposed loads were taken as the upper 5% fractile. The characteristic value of the shear strength of the soil was selected as $28,8^\circ$, one standard deviation below the mean (Schneider, 1977) except in the case of piles where the selected value was $30,4^\circ$, half a standard deviation below the mean. Partial factors were based on the National Annex to BS EN 1990:2002 and National Annex to BS

EN 1997:2004. The first step in the process was to find the “Eurocode-compliant” solution using characteristic values of loads and material properties. The solution obtained (minimum required value of B or L) is shown in the fourth column of Figure 1. Thereafter, the factor of safety was determined using unfactored values of the mean and characteristic values of the input parameters. The values obtained are given in Column 5 of Figure 1. Finally, the reliability index was determined using both FORM and Monte Carlo simulations (10^6 trials). Variable vertical actions and shear strength were assumed to be log-normally distributed, variable horizontal actions Gumbel distributed and soil density normally distributed. The reliability indices and the FORM design points are given in the final two columns of Figure 1. Further details of the analysis and an assessment of the variation in reliability indices and factor of safety with changes in parameters are given in De Koker and Day (2017). The reliability indices in Figure 1 show good agreement between the FORM and Monte Carlo methods indicating that FORM is suitable for use in many geotechnical problems. There is a remarkable consistency in the reliability indices (3,2 to 3,7) despite the different failure modes for the various examples. The factors of safety, however, varied widely between examples confirming that the factor of safety is a poor measure of reliability of a structure. Limit states design according to EN1997-1 is considered suitable for routine design in preference to working stress design methods.

4. RELIABILITY ASSESSMENT OF DA2*

Design approach DA2* is a variation of design approach 2 in which the actions on a footing are combined before they are factored (Frank et al, 2004), i.e. all components of the load vector attract the same partial action factor. This approach is favoured by the German design specification as it yields economical designs with similar levels of safety to previous design methods based on the global safety concept (Vogt & Schuppener, 2006). Simpson (2007) queried the acceptability of design approach DA2* as it requires significantly narrower strip footings compared to other design approaches when footings are subjected to vertical and horizontal loads. To assess the reliability of DA2* in comparison to design approach DA2 from EN1997-1, a similar procedure was followed to that described above, i.e. the width of a footing

required to resist various combinations of vertical and horizontal loads was determined

for both design approaches followed by a reliability analysis of the footing.

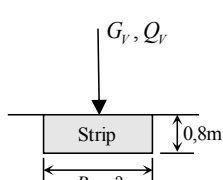
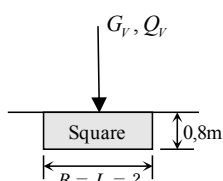
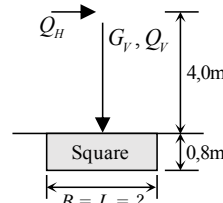
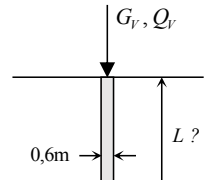
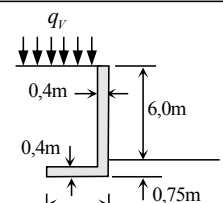
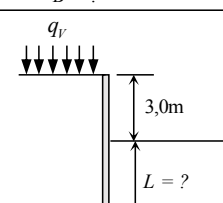
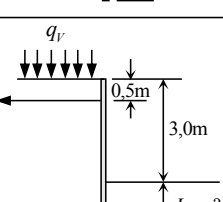
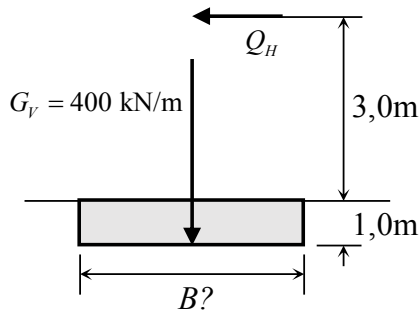
Example	Actions	Partial factors	Solution	Factor of safety	Reliability index	Design point
A 	$\overline{G}_V = 900 \text{ kN/m}$ $G_{Vk} = 900 \text{ kN/m}$ $\overline{Q}_V = 412.5 \text{ kN/m}$ $Q_{Vk} = 412.5 \text{ kN/m}$	$\gamma_G = 1,0$ $\gamma_Q = 1,3$ $\gamma_{\phi} = 1,25$ $\gamma_{\gamma} = 1,0$	$B = 3,10\text{m}$	$FoS_{char} = 2,50$ $FoS_{mean} = 5,18$	$\beta_{MC} = 3,409$ $\beta_{FORM} = 3,486$	$\phi' = 25,28^\circ$ $\gamma = 19,85 \text{ kN/m}^3$ $Q_V = 482,6 \text{ kN/m}$
B 	$\overline{G}_V = 3000 \text{ kN}$ $G_{Vk} = 3000 \text{ kN}$ $\overline{Q}_V = 1375 \text{ kN}$ $Q_{Vk} = 2000 \text{ kN}$	$\gamma_G = 1,0$ $\gamma_Q = 1,3$ $\gamma_{\phi} = 1,25$ $\gamma_{\gamma} = 1,0$	$B = L = 3,24\text{m}$	$FoS_{char} = 2,45$ $FoS_{mean} = 5,04$	$\beta_{MC} = 3,472$ $\beta_{FORM} = 3,497$	$\phi' = 25,27^\circ$ $\gamma = 19,83 \text{ kN/m}^3$ $Q_V = 1617 \text{ kN}$
C 	$\overline{G}_V = 3000 \text{ kN}$ $G_{Vk} = 3000 \text{ kN}$ $\overline{Q}_V = 1375 \text{ kN}$ $Q_{Vk} = 2000 \text{ kN}$ $\overline{Q}_H = 207 \text{ kN}$ $Q_{Hk} = 400 \text{ kN}$	$\gamma_G = 1,0$ $\gamma_Q = 1,3$ $\psi_0 = 0,7$ $\gamma_{\phi} = 1,25$ $\gamma_{\gamma} = 1,0$	$B = L = 3,89\text{m}$	$FoS_{char} = 2,55$ $FoS_{mean} = 6,54$	$\beta_{MC} = 3,617$ $\beta_{FORM} = 3,720$	$\phi' = 25,57^\circ$ $\gamma = 20,18 \text{ kN/m}^3$ $Q_V = 1333 \text{ kN}$ $Q_H = 626 \text{ kN}$
D 	$\overline{G}_V = 1200 \text{ kN}$ $G_{Vk} = 1200 \text{ kN}$ $\overline{Q}_V = 412.5 \text{ kN}$ $Q_{Vk} = 600 \text{ kN}$	$\gamma_G = 1,0$ $\gamma_Q = 1,3$ $\gamma_{\phi} = 1,0$ $\gamma_{\gamma} = 1,0$ $\gamma_{Rs} = 1,4$ $\gamma_{Rb} = 1,7$	$L = 16,1\text{m}$	$FoS_{char} = 1,56$ $FoS_{mean} = 2,20$	$\beta_{MC} = 3,208$ $\beta_{FORM} = 3,218$	$\phi' = 26,34^\circ$ $\gamma = 19,76 \text{ kN/m}^3$ $Q_V = 143,6 \text{ kN}$
E 	$\overline{q}_V = 6,875 \text{ kPa}$ $q_{Vk} = 10 \text{ kPa}$	$\gamma_Q = 1,3$ $\gamma_{\phi} = 1,25$ $\gamma_{\gamma} = 1,0$	$B = 3,52\text{m}$	$FoS_{char} = 3,12$ $FoS_{mean} = 6,88$	$\beta_{MC} = 3,301$ $\beta_{FORM} = 3,326$	$\phi' = 25,48^\circ$ $\gamma = 20,19 \text{ kN/m}^3$ $q_V = 14,95 \text{ kPa}$
F 	$\overline{q}_V = 6,875 \text{ kPa}$ $q_{Vk} = 10 \text{ kPa}$	$\gamma_Q = 1,3$ $\gamma_{\phi} = 1,25$ $\gamma_{\gamma} = 1,0$	$L = 4,00\text{m}$	$FoS_{char} = 1,63$ $FoS_{mean} = 2,34$	$\beta_{MC} = 3,351$ $\beta_{FORM} = 3,398$	$\phi' = 25,34^\circ$ $\gamma = 20,22 \text{ kN/m}^3$ $q_V = 7,17 \text{ kPa}$
G 	$\overline{q}_V = 6,875 \text{ kPa}$ $q_{Vk} = 10 \text{ kPa}$	$\gamma_Q = 1,3$ $\gamma_{\phi} = 1,25$ $\gamma_{\gamma} = 1,0$	$L = 2,57\text{m}$	$FoS_{char} = 1,25$ $FoS_{mean} = 1,43$	$\beta_{MC} = 3,237$ $\beta_{FORM} = 3,242$	$\phi' = 25,65^\circ$ $\gamma = 20,30 \text{ kN/m}^3$ $q_V = 7,29 \text{ kPa}$

Figure 1. Results of analysis for example problems

The strip footing chosen for this analysis is the same example used by Simpson (2007) and is shown in Figure 2. The required width of footing for both design approaches is shown in Figure 3a and the corresponding reliability

indices in Figure 3b. From Figure 3b, it is clear that the reliability the footing designed using DA2* is considerably lower than for design approach DA2 for high H_k/V_k .



Q_H variable,
 $CoV = 0.5$ (Gumbel)

$\bar{\phi}' = 36.1^\circ$, $CoV = 0.1$ (Log-normal)

$\bar{\gamma} = 19 \text{ kN/m}^3$, $CoV = 0.05$ (Normal)

$\phi'_k = 32.5^\circ$

$\gamma_k = 19 \text{ kN/m}^3$

Figure 2. Strip footing example, DA2*

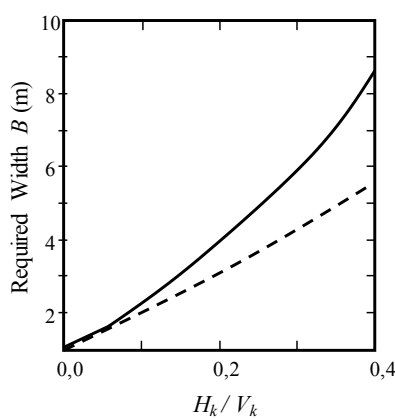


Figure 3a. Required footing width

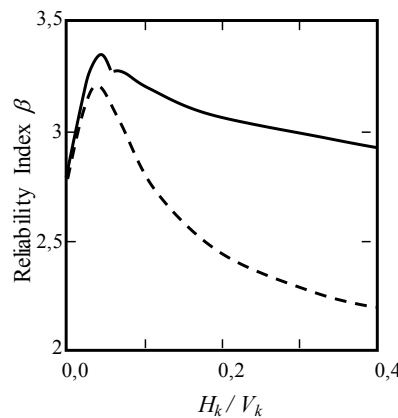


Figure 3b. Reliability index

Legend:

- DA2:
 $\gamma_{G_{fav}} 1.0$, $\gamma_{Q_H} 1.5$
 $\gamma_{G_{unfav}} 1.35$, $\gamma_{Q_H} 1.5$
 $\gamma_\phi 1.0$, $\gamma_R 1.4$
- - - DA2*:
 $\gamma_{G_{fav}}$ and $\gamma_{Q_H} 1.0$
 $\gamma_{G_{unfav}}$ and $\gamma_{Q_H} 1.35$
 $\gamma_\phi 1.0$, $\gamma_R 1.4$

5. CONCLUSIONS

The First Order Reliability Method (FORM) is sufficiently accurate and easy to apply to justify its use in a geotechnical design office. The major problem remains obtaining sufficient data to justify the choice of input parameters. Limit states design is acceptable for routine design purposes.

DA2* gives significantly lower levels of reliability for highly eccentric loads than DA2.

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