HYDRAULIC ANALYSIS OF THE WATER HAMMER IN BRANCH WATER SUPPLY NETWORK DEPENDING ON BRANCH DISTANCE

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It is generally known that water supply systems consist of the following basic components: water source, water treatment plant, reservoirs, supply and main pipelines, and water supply network.

Depending on the terrain configuration and users distribution, the water supply network can usually be of branch type, loop type or combined. The water supply network in the urban areas is mostly a loop type network, whereas in the rural - outskirt areas, there is a branch type network.

Subject of this hydraulic analysis is to comprehend the influence of branch density in the branch water supply network on the size and distribution of water hammer in the network.

Keywords: water supply network, water hammer, method of characteristics

1. INTRODUCTION

The large number of components fitted in one water supply system which should operate continuously, presents a complex system which, due to its complexity, most frequently in practice the hydraulic analysis is made for assumed quasi-stationary regime of flow in systems under pressure. However, in reality the water supply systems are systems in which there are constant changes in pressure and flow, that is, the flow is non-stationary and in case of the need for considering the state in the water supply network when the non-stationary regime of flow occurs, it is advisable for the hydraulic analysis to be made for non-stationary flow in systems under pressure [1].

Size, distribution, form, and duration of the changes in pressure at non-stationary regime in one water supply system cannot be determined with a simple and unique mathematical model for analysis of the water hammer. That is due to the fact that water supply systems are complex systems and practically each water supply system represents a separate system for hydraulic analysis.

The characteristics of the hydraulic impact in such complex systems depend on a large number of parameters [9] such as:

- Type of the initiator of the immediate change in pressure and flow
- Location of the initiator of the immediate change in pressure and flow
- Current pressure and flow in all lines of the water supply network
- Material from which the water pipes are made
- Density of lines – branches of the water supply network
- Length of lines – branches of the water supply network
- Height configuration of the water supply network.

According to the previously stated, for the hydraulic analysis of water hammer occurrence subject of analysis in this thesis, the mathematical model HTM (Hydraulic Transient Model) aimed at analysis of such systems will be used.

2. BASIC EQUATIONS FOR WATER HAMMER

According Wylie [7] (1993), the water hammer is defined as the hydraulic variable occurrence of flow, which causes an increase of overpressure in a pipeline system. The water hammer can be generated by certain operational measures such as: opening or closing of the valve, turning the pumps on or off, abrupt cracking of the tube etc.

Starting points in the mathematical description of the water hammer [11] are the basic laws in the mechanics of fluids:

- Law of maintaining the amount of movement and
- Law of maintaining weight.

Satisfying these basic principles for conservation/maintenance comes to the dynamic equation and the equation of continuity.

The final form of the dynamic equation for unsteady flow in closed systems under pressure:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial \Pi}{\partial x}+\frac{\lambda}{2D}V|V|=0 \qquad (1)$$

The convective acceleration V∂V/∂x or acceleration along the pipe is significantly lower compared to the acceleration ∂V/∂t or acceleration over time, so mostly that convective acceleration is overlooked and the dynamic equation is written:

$$\frac{\partial V}{\partial t}+g\frac{\partial \Pi}{\partial x}+\frac{\lambda}{2D}V|V|=0 \qquad (2)$$

Assuming that the density of the fluid changes very little in terms of piezometric height (ρ= const), the equation of continuity gets the following form:

$$V\frac{\partial \Pi}{\partial x}+\frac{\partial \Pi}{\partial t}-V\sin\alpha+\frac{a^2}{g}\frac{\partial V}{\partial x}=0 \qquad (3)$$

Where a is the speed of propagation of the pressure wave and it is determined by the ratio of compression of the fluid and the module of elasticity of the tube:

$$a=\sqrt{\frac{K}{\rho\left(1+\frac{K}{E}\frac{D}{e}c_1\right)}} \qquad (4)$$

The coefficient c1 depends of the pipe anchorage and is equal to:

- c1=1–μ/2 – pipe anchorage only at the upstream
- c1=1–μ2 – pipe anchorage throughout against axial movement
- c1=1 – pipe anchorage with expansion joints throughout.

3. METHOD OF CHARACTERISTICS FOR SOLVING BASIC EQUATIONS OF WATER HAMMER

With the method of characteristics [17] the basic partial differential equations which are not integrable in closed form, are transformed into ordinary differential equations which have a solution in a closed form. The basic equations, the equation of continuity and the dynamic equation can be designated with L_1 и L_2:

$$L_1=g\frac{\partial \Pi}{\partial x}+\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{\lambda}{2D}V|V|=0 \qquad (5)$$

$$L_2=\frac{\partial \Pi}{\partial t}+V\frac{\partial \Pi}{\partial x}+\frac{a^2}{g}\frac{\partial V}{\partial x}-g\sin\alpha=0 \qquad (6)$$

These linear equations can be combined as follows:

$$L = L_1 + \chi L_2 \tag{7}$$

The two dependent variables, the speed V and the pressure Π are in a function from the position and time, V=V(x,t) и Π=Π(x,t). The material statements of these dependent variables are total accelerations which are determined by the convective and local acceleration:

$$\frac{d\Pi}{dt} = \frac{\partial \Pi}{\partial x}\frac{dx}{dt} + \frac{\partial \Pi}{\partial t} \tag{8}$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial t} \tag{9}$$

Comparing the expression of the convective acceleration of equation (7) to those of equations (8) and (9), follows:

$$\frac{dx}{dt} = V + \frac{g}{\chi} = V + \frac{\chi a^2}{g} \tag{10}$$

Then equation 7 is written:

$$\chi \frac{d\Pi}{dt} + \frac{dV}{dt} + \frac{\lambda}{2D} V|V| - \chi g \sin\alpha = 0 \tag{11}$$

The solution of equation 11 is:

$$\chi = \pm \frac{g}{a} \tag{12}$$

$$\frac{dx}{dt} = V \pm a \tag{13}$$

From the previous equation it can be concluded that it's about two families of curves that are practically straight lines, where the speed of propagation is constant and many times faster than the basic flow, so the system of two partial differential equations are transformed into system of ordinary four differential equations which are marked with a C+ and C– and determine straight lines:

$$\left.\begin{array}{l} \frac{d\Pi}{dt} + \frac{a}{g}\frac{dV}{dt} + \frac{\lambda}{2D} V|V| - V \sin\alpha = 0 \\ \frac{dx}{dt} = V + a \end{array}\right\} C^{+} \tag{14}$$

$$\left.\begin{array}{l} -\frac{d\Pi}{dt} + \frac{a}{g}\frac{dV}{dt} + \frac{\lambda}{2D} V|V| + V \sin\alpha = 0 \\ \frac{dx}{dt} = V - a \end{array}\right\} C^{-} \tag{15}$$

4. NUMERICAL MODEL

Figure 1 shows discretization of the physical system in a numerical network with computing steps Δx and Δt where the solutions are obtained at the intersection of the positive and negative lines of characteristics [8], [9], [10], [11].

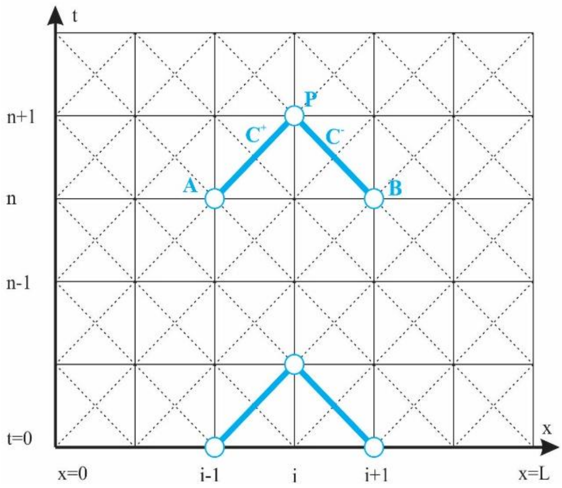

Figure 1. Numerical network for the method of characteristics

According the given numerical network, equations (14) and (15) can be written as follows:

$$\frac{d}{dt}\left(\Pi \pm \frac{a}{g} V\right) + \lambda \frac{a}{D}\frac{V|V|}{2g} \mu V \sin\alpha = 0 \tag{16}$$

The previous equation can be integrated along the positive and negative characteristics, i.e. along the length of the lines AP and BP, as follows:

$$\int_{tA}^{tP} \frac{d}{dt}\left(\Pi + \frac{a}{g} V\right) dt + \int_{tA}^{tP} \left(\lambda \frac{a}{D}\frac{V|V|}{2g} - V \sin\alpha\right) dt = 0 \tag{17}$$

$$\int_{tB}^{tP} \frac{d}{dt}\left(\Pi - \frac{a}{g} V\right) dt + \int_{tB}^{tP} \left(\lambda \frac{a}{D}\frac{V|V|}{2g} + V \sin\alpha\right) dt = 0 \tag{18}$$

After integration, equations of positive and negative characteristic are written:

$$\frac{\Pi_P - \Pi_A}{\Delta t} + \frac{a}{g}\frac{V_P - V_A}{\Delta t} + \frac{\lambda a}{2gD} V_A |V_A| - V_A \sin\alpha = 0$$

$$\frac{\Pi_P - \Pi_B}{\Delta t} - \frac{a}{g}\frac{V_P - V_B}{\Delta t} + \frac{\lambda a}{2gD} V_B |V_B| - V_B \sin\alpha = 0$$

If it is known that the hydraulic analysis is important to determine the change in the flow and height position of the hydrodynamic line in any section along the pipe and at a specified interval, additional approximating is introduced that the cross-section of the pipe throughout its length is constant and if is known that the average speed can be determined by the equation *V=Q/A*, the previous equations knowing the numerical network can be written in the following form:

$$\Pi_i^{n+1} - \Pi_{i-1}^n + \frac{a}{gA}\left(Q_i^{n+1} - Q_{i-1}^n\right) + \frac{\lambda\Delta x}{2gDA^2} Q_{i-1}^n \left|Q_{i-1}^n\right| - \frac{\Delta t}{A} Q_{i-1}^n \sin\alpha = 0 \quad (19)$$

$$\Pi_i^{n+1} - \Pi_{i+1}^n - \frac{a}{gA}\left(Q_i^{n+1} - Q_{i+1}^n\right) - \frac{\lambda\Delta x}{2gDA^2} Q_{i+1}^n \left|Q_{i+1}^n\right| - \frac{\Delta t}{A} Q_{i+1}^n \sin\alpha = 0 \quad (20)$$

If:

$$B = \frac{a}{gA} \quad \text{and} \quad M = \frac{\lambda\Delta x}{2gDA^2}$$

Using the previous equations, for the pressure, i.e. for the height position of the hydrodynamic line, it can be written:

$$\Pi_i^{n+1} = \Pi_{i-1}^n - B\left(Q_i^{n+1} - Q_{i-1}^n\right) - MQ_{i-1}^n\left|Q_{i-1}^n\right| + \frac{\Delta t}{A} Q_{i-1}^n \sin\alpha = 0 \quad (21)$$

$$\Pi_i^{n+1} = \Pi_{i+1}^n + B\left(Q_i^{n+1} - Q_{i+1}^n\right) + MQ_{i+1}^n\left|Q_{i+1}^n\right| + \frac{\Delta t}{A} Q_{i+1}^n \sin\alpha = 0 \quad (22)$$

If the parameters of flow are known in the time interval (n), then we get:

$$\Pi_i^{n+1} = CP - BQ_i^{n+1} \quad (23)$$

$$\Pi_i^{n+1} = CM + BQ_i^{n+1} \quad (24)$$

Where:

$$CP = \Pi_{i-1}^n + BQ_{i-1}^n - MQ_{i-1}^n\left|Q_{i-1}^n\right| + \frac{\Delta t}{A} Q_{i-1}^n \sin\alpha$$

$$CM = \Pi_{i+1}^n - BQ_{i+1}^n + MQ_{i+1}^n\left|Q_{i+1}^n\right| + \frac{\Delta t}{A} Q_{i+1}^n \sin\alpha$$

In the previous equations the article which include the slope of the pipes (sinα) is very small and often overlooked, so the equations (25) and (26) are written:

$$CP = \Pi_{i-1}^n + BQ_{i-1}^n - MQ_{i-1}^n\left|Q_{i-1}^n\right| \quad (25)$$

$$CM = \Pi_{i+1}^n - BQ_{i+1}^n + MQ_{i+1}^n\left|Q_{i+1}^n\right| \quad (26)$$

From equations (25) and (26) is obtained a basic equation of the characteristics for determining the elevation of the hydrodynamic line:

$$\Pi_i^{n+1} = \frac{CP + CM}{2} \quad (27)$$

Knowing the piezometric height (Πi) in the time period (n+1), the flow (Qi) is determined by equations (25) and (26).

5. BOUNDARY CONDITIONS

The conditions of the flow that govern within the boundary of the system under pressure – the water supply system are defined as boundary conditions. Their definition is of crucial importance for getting the solution at the points in the system. Follow-on are the most common cases of boundary conditions encountered in the water supply systems [5].

Serial connection of two pipes in a junction

Pressure: $\Pi_{1,N}^{n+1} = \Pi_{2,1}^{n+1} = \Pi^{n+1}$

Flow: $Q_{1,N}^{n+1} = Q_{2,1}^{n+1} = Q^{n+1} = \frac{CP_1 - CM_2}{B_1 + B_2}$

Connection of more pipes in a junction

Pressure: $\Pi^{n+1} = \Pi_{1,N}^{n+1} = \Pi_{2,1}^{n+1} = \Pi_{3,1}^{n+1} = \Pi_{4,1}^{n+1}$

$$\Pi^{n+1} = \frac{CP_1/B_1 + CM_2/B_2 + CM_3/B_3 + CM_4/B_4}{1/B_1 + 1/B_2 + 1/B_3 + 1/B_4}$$

Flow: $-Q_{1,N}^{n+1} = \frac{\Pi^{n+1}}{B_1} - \frac{CP_1}{B_1}$; $Q_{2,1}^{n+1} = \frac{\Pi^{n+1}}{B_2} - \frac{CM_2}{B_2}$

$Q_{3,1}^{n+1} = \frac{\Pi^{n+1}}{B_3} - \frac{CM_3}{B_3}$; $Q_{4,1}^{n+1} = \frac{\Pi^{n+1}}{B_4} - \frac{CM_4}{B_4}$

Reservoir at the end of pipeline

Pressure: $\Pi_1^{n+1} = \Pi_R$

Flow: $Q_1^{n+1} = \frac{\left(\Pi_1^{n+1} - CM\right)}{B}$

Valve at the end of the pipeline

Pressure: $\Pi_1^{n+1} = CP - BQ_N^{n+1}$

Flow:

$$Q_N^{n+1} = -B \cdot C_1 + \sqrt{(B \cdot C_1)^2 + 2C_1(CP - Z_Z)}$$

$$C_1 = g\frac{A_C^2}{\xi_Z}$$

Valve at the middle of the pipeline

Pressure:

$$\Pi_{1,N}^{n+1} = CP_1 - B_1 Q_{1,N}^{n+1} \; ; \; \Pi_{2,1}^{n+1} = CM_2 - B_2 Q_{2,1}^{n+1}$$

$$CP_1 - B_1 Q^{n+1} - CM_2 - B_2 Q^{n+1} - C_1 Q^{n+1} \left| Q^{n+1} \right| = 0$$

If the condition is completed CP1–CM2>0 than follows:

Flow:

$$Q^{n+1} = \frac{-(B_1 + B_2) + \sqrt{(B_1 + B_2)^2 + 4C_1(CP_1 - CM_2)}}{2C_1}$$

If CP1–CM2< 0 than follows:

Flow:

$$Q^{n+1} = \frac{(B_1 + B_2) - \sqrt{(B_1 + B_2)^2 - 4C_1(CP_1 - CM_2)}}{2C_1}$$

6. HYDRAULIC ANALYSIS OF THE BRANCH TYPE WATER SUPPLY NETWORK BY MEANS OF THE HTM – HYDRAULIC TRANSIENT MODEL

Calculation template of zone gravitation water supply system of the branch type water supply network is used for the development of the mathematical model HTM, in which the water from the water source is firstly distributed in the upper elevation zone and the water excess, i.e. the needs for water in the lower zone fill the reservoir for the lower zone. According to the previously stated, it can be concluded that there is no reservoir space for the upper zone, and it can be said that the provided reservoir, in addition to leveling the supply amounts of water including the needs of water for the lower zone, also represents an interruption chamber.

Hydro-mechanical equipment for regular operation of the entire water supply system is provided in the reservoir for the lower zone. Namely, there is a pressure control valve placed before the reservoir which sustains the input pressure in the reservoir (Pressure sustaining valve) to prevent the decrease of the water pressure in the water supply network of the higher zone, i.e. not to cause fall of the hydrodynamic line to the reservoir peak elevation. After the pressure control, there is a valve by which the level in the reservoir is regulated. The valve in this calculation pattern is practically the initiator for causing non-stationary flow. Namely, in the moment when the reservoir reaches the maximum peak elevation, the valve immediately closes, and vice versa, in the moment when the level in the reservoir drops to a certain limit, the valve opens suddenly.

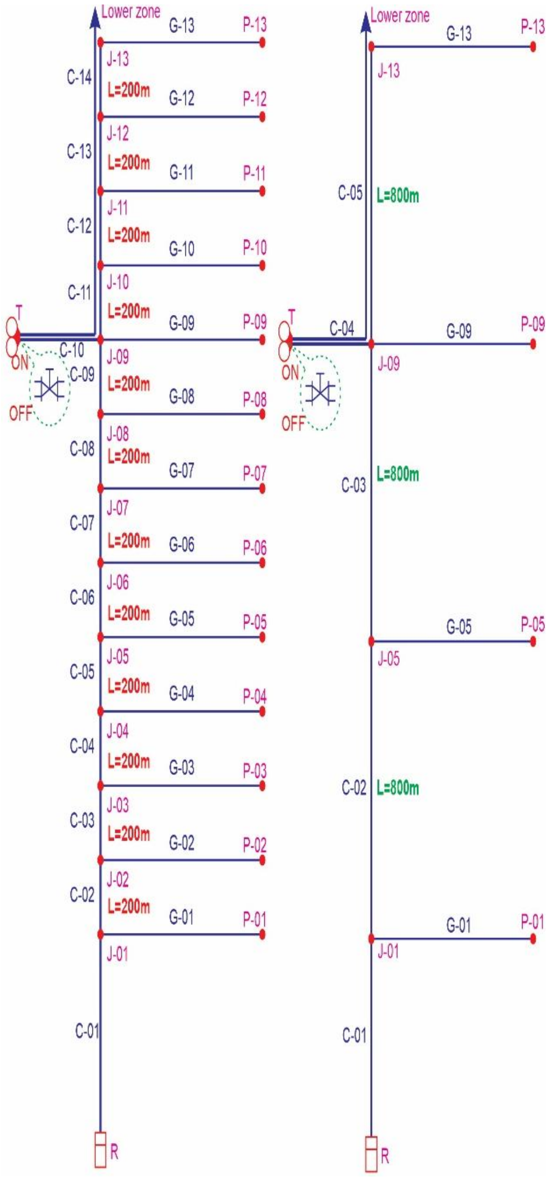

Figure 2. Calculation scheme for the development of the mathematical model HTM

The choice for such calculation pattern (Figure 2) is made in order to analyze the phenomenon of the water hammer in a complex system with gravitation inflow and closing valve at the end of a primary pipeline. This calculation pattern is practically an equivalent to the model of simple system - a reservoir and a pipeline having a valve at the end. Two scenarios are analyzed:

I. First scenario. Branch connections of the secondary network are at a distance of 200 m, and all the pipes of the water supply network are of the same material - ductile pipes.

II. Second scenario. Branch connections of the secondary network are at a distance of 800 m, and all the pipes of the water supply network are of the same material - ductile pipes.

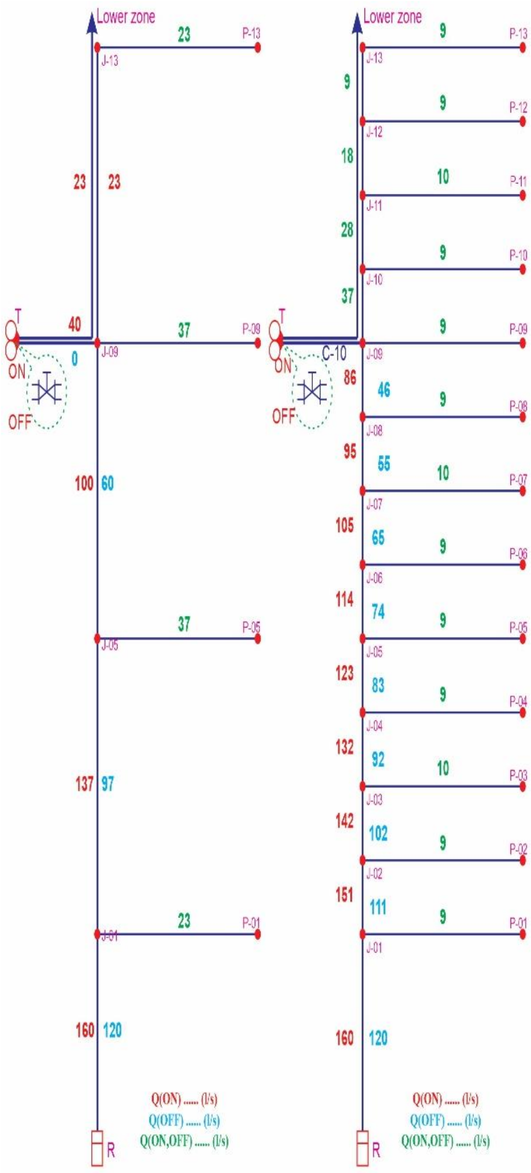

Figure 3. Water flow per pipeline at stationary regime – Scenario I

Table 1 shows the pipeline material characteristics and water as fluid.

Table 1. Material characteristics of pipelines and water as fluid

Characteristics	Value
DUCTIL PIPES	
Young modulus of elasticity	170 GPa
Poisson factor	0.28
FLUID – WATER	
Temperature	20°C
Density	998 kg/m^3
Modulus of elasticity	2.19 GPa
Kinematic viscosity	1.01×10^{-6} m^2/s

The hydraulic analysis in the water supply system at stationary regime of flow in the system is made in the software package WaterGEMS, and Figure 3 shows running quantities of water per line at stationary regime which are input parameters – initial conditions in the mathematical model HTM for the hydraulic impact analysis.

7. RESULTS OF HYDRAULIC ANALYSIS

In addition to the following figures, there is a graphic presentation of the output results of the hydraulic analysis for branch type water supply network of different branch density in hydraulic impact occurrence at sudden valve closure.

Important points subject to analysis are:

- Place of the non-stationarity initiator – valve (junction "T")
- The first junction where the upper zone J-01 users are connected, and
- Final junction of the upper zone J13
-

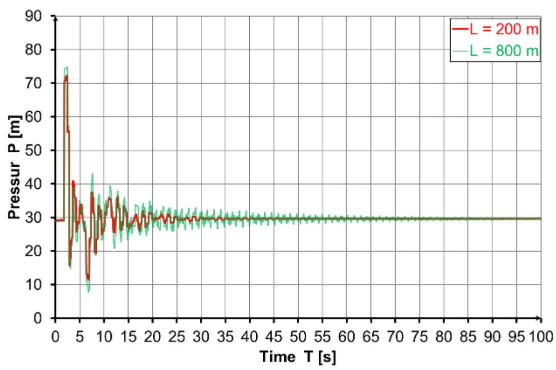

Figure 4. Pressure distribution at junction T, quick closing valve

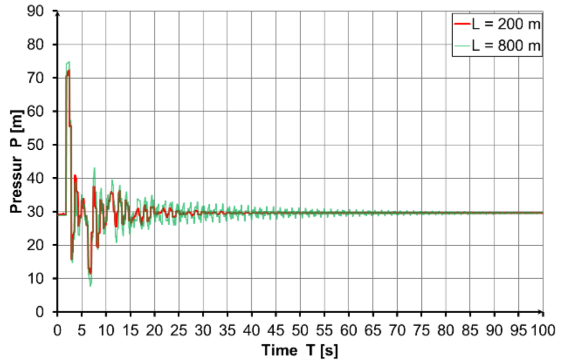

Figure 5. Pressure distribution at junction J-01, quick closing valve

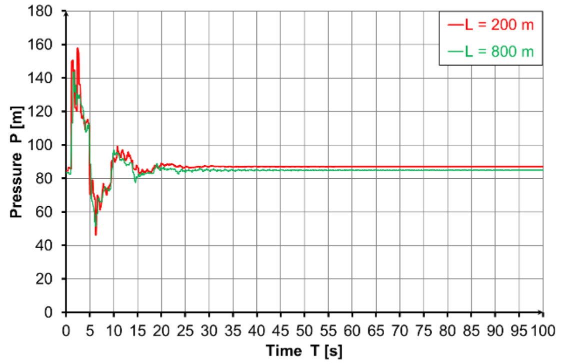

Figure 6. Pressure distribution at junction J-13, quick closing valve

8. CONCLUSION

From the results obtained, it can be observed that there is a significant increase in the pressure both at the initiation place of non-stationarity and in the water supply network itself. According to that, a recommendation can be made that for such or similar systems where occurrence of non-stationarity in the water supply system is expected, they need to be analyzed as a system under pressure with non-stationary flow.

From the analyzed non-stationarity, it can be concluded that pressure increase at the non-stationarity initiation place in knot "T" is identical in both analyzed cases. However, the non-stationarity duration in water supply systems of denser branches is much shorter compared to the other systems. Considering the main pipeline – pipeline connecting the water catchment and the reservoir along which the branches "J-01" are connected, in the systems of denser branches the amount of above-pressure is lower compared to the remaining system, and also the non-stationarity duration in denser branches is shorter compared to the other ones. While the situation is different at the ends of the branches J-13, i.e. the water supply system of denser branches has larger increase of the above-pressure, and the non-stationarity duration is identical, this is due to the fact that the amount of water flowing through the branch in denser branches is much lower.

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