DATA PROCESSING FROM PRECISE LEVELLING IN SEISMIC ACTIVE REGIONS

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This paper presents the processing of the data from the precise levelling measurements in the seismic active area of the Skopje valley. Precise levelling results are affected by the Earth's gravity field, to eliminate the effect of the gravity field gradient, corrections need to be applied in processing of precise levelling data. For this purpose the processing of levelling data is performed together with GNSS coordinates and gravity data, which are acquired on a part of the state levelling network from first order, located on the territory of the City of Skopje. They are used for transformation of the height differences in the system of dynamic, orthometric and normal heights. Also other corrections are applied to observed levelling data to minimize the effects of known systematic errors.

Key words: precise levelling, gravity, corrections, data processing, orthometric, height systems.

1. INTRODUCTION

If the height differences between points A, B, and C (Figure 1) would be determined only by applying a geometric levelling, then the results of the levelling, which previously are not considered to be laden by accidental and systematic errors, would indicate the following:

- Between points A and B, which are on the same level surface, there is a height difference,
- The value of leveled height differences depends on the path of levelling,

$$\int_A^C dh \neq \int_B^C dh, \tag{1}$$

- The sum of height differences in a closed polygon is not zero.

$$\oint dh \neq 0, \text{ i.e. } \sum_i \Delta h_i \neq 0, \tag{2}$$

In order to eliminate these effects, it is necessary to introduce the influence of the gravity force in the results of the measurements. [1]

As it is well known, the Earth's gravity varies according to the locations of observation points. The gravity field is stronger on the poles and weaker on the equator due to the centrifugal effect of the rotating Earth. The variations in the gravity field are also due to the heterogeneous nature of the Earth's interior and crust.

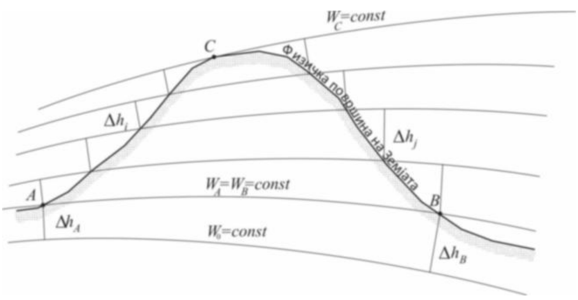

Figure 1. Dependence of the results of the levelling from the levelling path [1]

The processing of the results from the measurements in the levelling network of high accuracy has been carried out in two parts. [1]

The first part of the processing involved:

- Checking the quality of the measured data, in which the conditions defined by the parameters for controlling and monitoring the measurements are independently checked,
- Entering corrections in all height differences for the influence of the mean meter of the relevant pair of rods,
- Transformation of the height differences in the system of geopotenital, dynamic, orthometric and normal heights.

The second part of the processing involved the adjustment of the height differences in all listed height systems, with the fundamental benchmark in Skopje - FRSK adopted for the network's datum.

2. HEIGHT SYSTEMS

The ideal height system must meet the following conditions:

- The height of the points must be unique (they should not depend on the levelling direction),
- In closed polygons, height differences must be determined in such a way that their sum is zero (the closure of the polygon when determining the height systems must be eliminated),
- The heights must be determined only on the basis of the measurements of the physical surface of the Earth without the introduction of hypotheses,
- The reference surface of the heights must be physically clearly defined,
- The heights must have a unit of measure,
- The heights must have a geometric interpretation,
- The heights must be able to connect with the height networks of neighboring countries.

All the aforementioned requirements cannot be provided simultaneously because the first two conditions are fulfilled only when the differences in the Earth's gravity are used to determine the height of the points. In that case, the heights do not have a geometric interpretation nor have a unit of measure.

In this direction, a number of height systems are proposed which can satisfy only one of the quoted conditions, most often used are the following: geopotential, orthometric, normal, dynamic and spheroidal heights.

2.1 GEOPOTENTIAL NUMBERS

For the unambiguous determination of the height of the points, regardless of the path of levelling, the potential difference is used.

Practically the geopotential numbers are using the results of the measurements of the acceleration of the gravity force and the levelling for computing the line integral:

$$C_{P_i} = dW = -\int_{P_i}^{P_0} g \cdot dh = \int_{P_0}^{P_i} g \cdot dh \approx \sum_{k=P_0}^{P_i} g_k \cdot \Delta h_k \qquad (3)$$

Units of the geopotential numbers are the units of potentials, m^2s^2. The same year, when it was proposed by the International Association, it was decided that the geopotenital height will be with unit **1 g.p.u = 1 kgal • m**, where g.p.u. is shortcut for a *geopotential unit*.

The heights determined with the use of geopotential numbers are not suitable for practical application because they do not have a geometric interpretation, and the unit of geopotential numbers is not a unit of length. For these reasons, the physical heights for practical applications are defined in another way. [1]

If the value of the geopotential number is divided by the contracted value of the acceleration of the gravity force of the earth gravity (G),

$$H = \frac{C}{G}, \quad (4)$$

a simple transformation of the geopotential number in metric heights will be performed, and the characteristics of the geopotential number will be retained.

Different G values define different *heights systems*.

2.2 DYNAMIC HEIGHTS

The height difference in the system of dynamic heights can be calculated in two ways:

- Directly from the difference of geopotential numbers,
- By calculating dynamic correction.

Dynamic correction and dynamic height difference is obtained as:

$$\Delta H_{DYN} = \Delta H + DC, \quad (5)$$

$$DC = \frac{g - \gamma_0}{\gamma_0} \cdot \Delta H, \quad (6)$$

where *DC* is dynamic correction and γ_0 is normal gravity.

For normal gravity γ_0 it is necessary to adopt a value calculated for the 45^0 latitude. Calculating the value of the normal gravity is to be done by using the expression associated with GRS80 geodetic reference system. [8]

2.3 ORTHOMETRIC HEIGHTS

Orthometric height of the point P is called the vertical section from point P to the geoid (Figure 2)

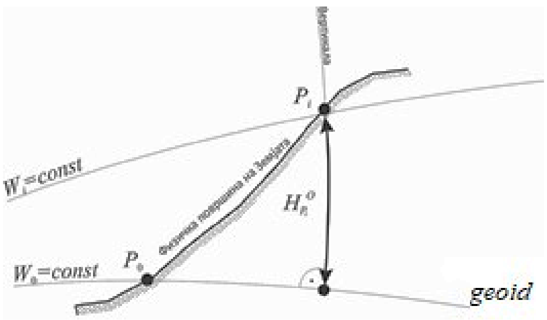

Figure 2. Orthometric heights [1]

Orthometric heights are calculated by the expression:

$$H_P^O = \frac{C_P}{\bar{g}_P} \quad (7)$$

where the C_P is the geopotential number of the point P, and $\bar{g}_P$ the mean value of the acceleration of the earth's gravity of the vertical between the points P_0 and P. [9]

$$OP_{1,2} = H^O_{P_1} \cdot \frac{\bar{g}_{P_1} - G_0}{G_0} - H^O_{P_2} \cdot \frac{\bar{g}_{P_2} - G_0}{G_0} + \sum_{k=P_1}^{P_2} \frac{g_k - G_0}{G_0} \Delta h_k \quad (8)$$

Above is the formula for calculating the orthometric correction. Points that have the same orthometric heights are not on the same real level surface, since it follows that the orthometric heights simulate fall.

Poincare Prey reduction

The value of the acceleration of Earth force gravity on the vertical segment from P to P_0 is not known and cannot be accurately determined neither by measurement nor by calculation. For the practical realization of the orthometric heights it is necessary to approximate with the aid of hypotheses about the mass distribution of the part of the earth's crust, between the physical surface of the Earth and the geoid. [1]

One of the ways to calculate the inner value of the acceleration of the force of the earth's gravity is by applying the Poincare Prey reduction.

$$g_Q \approx g_P - 0.84807 \cdot 10^{-6} \cdot (H_Q - H_P) \quad (9)$$

The Poincare-Prey reduction does not take into account the influence of the topographic masses on the acceleration of the force of the earth's gravity (i.e. it neglects the influence of the masses that are over and the impact of the deficit within the Bouguer's plane).

2.4 NORMAL HEIGHTS

For the calculation of the normal height difference it is necessary to apply the so-called normal correction, indicated below with f on the obtained average height difference.

The normal correction is also called gravimetric correction and reflects the impact of the gravitational field or the non-parallelness of the level surfaces along the line levelling route.

The normal correction to the individual height difference is a sum of two parts: [2]

$$f_{i,i+1} = I_{i,i+1} + II_{i,i+1}, \tag{10}$$

where:

$$I_{i,i+1} = -\frac{1}{\gamma_m}(\gamma_0^{i+1} - \gamma_0^{i})H_m \tag{11}$$

It's called *first normal correction,*

$$II_{i,i+1} = \frac{1}{\gamma_m}(g - \gamma^N)_m h_{i,i+1} \tag{12}$$

It's called *second normal correction.*

The formula set for the first normal correction in the Level I and II Level Instruction, GUGKK, 1980 is as follows:

$$I = -\frac{\beta}{\rho'} sin2\varphi\Delta\varphi H_m\ [mm], \tag{13}$$

According to the Level I and II instructions, the gravimetric data and the method of processing must guarantee an error in determining the normal correction not greater than 0.05 mm and with exception in the high parts 0.10 mm. [2]

3. CORRECTIONS APPLIED TO PRECISE LEVELLING OBSERVATIONS

To achieve the highest degree of accuracy in the measurement of elevation differences corrections must be applied to precise levelling observations. Observational procedures have also been designed to provide the most effective method to acquire data. In addition, those systematic errors which cannot be sufficiently controlled by instrumentation or observational techniques are minimized by applying appropriate corrections to the observed data. [4]

Rod scale correction

Precise levelling rods should be calibrated before and after each project if practical, whenever possible damage has occurred, or at least once during each year of use. The calibrated length of a rod is usually determined by comparing its invar strip to a standard meter. The length excess of an average rod meter is computed from the "actual minus nominal" length differences observed at several points along the rod. The correction for the pair of rods mean meter is entered in the levelling results derived by concrete levelling rods. This correction ensures a uniform scale.

Level collimation correction

The effects of the collimation error of a levelling instrument are best minimized by field procedures. If sight lengths are balanced, i.e., DS = 0 and SDS = 0, where DS is the difference between backward and forward sight lengths at one setup (DS = backsight distance - foresight distance) and SDS is the accumulated DS for a section, the effect of the collimation error approaches zero. A well-adjusted instrument also minimizes this error without balancing sight lengths, although the collimation error of most levelling instruments changes slightly throughout the day as a result of changing temperature. A test for checking the levelling instrument collimation error is the two peg test, which must be carry out before starting the measurements.[4]

Refraction correction

The most suitable formula for the correction of the height differences, arising from the vertical refraction, is a simplified version of the model developed by Professor T. J. Kukkamaki of the Finnish Geodetic Institute (Kukkamaki 1939).[6]

The vertical temperature gradient in the ground aerial layer determines the degree of influence of the refraction on the results of the geometric levelling. Temperature measurement levels are suggested by Kukkamaki, 1938 at heights of 0.3 m, 0.9 m and 2.7 m on the rods or the ratio between two adjacent heights is 1/3. [3]

3.1 ASTRONOMIC CORRECTION

The astronomic correction is applied to account for the effect of tidal accelerations due to the Moon and Sun on the Earth's equipotential surfaces. The astronomic correction is small, amounting, at most, to 0.1 mm/km, but it accumulates in the north-south direction.

The required input for the correction is: time and date of measurements, heights of the benchmarks, and geodetic positions of the "From" and "To" bench marks. [4]

Tidal correction to normal height differences

The determination of normal height differences in the zero-tidal system, according to EUREF, 2008:

$$H_2(\varphi) = -295.41\sin^2\varphi - 0.42\sin^4\varphi + 99.40 \ [\mathrm{mm}] \quad (14)$$

Add the value computed from the formula above to normal heights in the mean tidal system to get normal heights in the zero tidal system.

Tidal correction to geopotenital numbers

$$W_2(\varphi) = -288.41\sin^2\varphi - 1.95\sin^4\varphi + 97.22 \quad [\mathrm{mgpu}] \quad (15)$$

Add the value computed from the formula above to geopotential numbers in the mean tidal system to get geopotential numbers in the zero tidal system. [7]

4. DATA PROCESSING

In the data processing for the precise levelling measurements the following data is used as input data:

- The results of the measurements of the height differences shown in the levelling field book,
- Gravity data for the levelling benchmarks,
- Terrain forms in which are described benchmarks location and coordinates in ETRS89 system. [2]

4.1 METHODS OF MEASURMENT AND USED INSTRUMENTS

Levelling measurements

For the measurement of height differences the precise digital level **Leica DNA 03** is used, equipped with two three metre barcoded levelling rods with invar tape, with original holders and metal slippers. The instrument has a declared accuracy of **0.3 mm/km.** During the measurement, readings and registration of the temperature values were performed. With the applied measurement methodology and the levelling instrument used, the relative accuracy of determining the height difference is better than **+/- 1 mm • km**$^{1/2}$. [5]

Gravimetric measurements

The value of the acceleration of the force of gravity (g) at the sites where the benchmarks are stabilized is determined by applying relative gravimetric measurements. Gravimetric measurements are performed using the method of profiles using gravimeters of the type Scintrex CG3 and Scintrex CG5, by performing 3 cycles of measurements of 60 seconds. The achieved accuracy of determining the value of the acceleration of the force of the gravity is +/- 60 microgals.

GNSS measurements

The position of the benchmarks was determined by three series of RTK measurements in relation with MAKPOS or with static method with time period of 30'. The coordinates of the benchmarks were determined in the European ETRS 89 coordinate system with an accuracy of +/- 1 to 3 centimeters, and then transformed into the Macedonian state coordinate system with an accuracy of about +/- 10 centimeters.[1]

Based on the values obtained form the forward-backward levelling is calculated "average height differences" column. (Table 1) The values of both forward (I) and backward (II) levelling columns are corrected by a correction for rod scale defined in a certified laboratory also known as "*mean meter of the pair of rod* ". From the corrected height differences average values are calculated. (Table 2)

Table 1. Measured height differences [5]

Nº BM	lm between BM, S, km	Height difference, m		d=II - I mm	Average value of height differences
		I	II		
FRSK V2-R1	2.01	28.53262	-28.53317	-0.55	28.53290
V2-R1 V2-R2p	2.18	20.9649	-20.96575	-0.85	20.96533
V2-R2p V2-R3	1.36	-14.1584	14.15921	0.84	-14.15879
FRSK L17-R1	1.65	-4.51362	4.51348	-0.14	-4.51355
L17-R1 L17-R2	1.95	-2.69487	2.69573	0.86	-2.69530

Table 2. Corrected height differences for mean meter of the pair of rods

L17_R2 - V2_R3

No	Mean rod meter correction			Diff. d [mm]	Allowed D [mm]
	I	II	Average		
	-1.000128	-1.000128			
L17_R1-R2	2.69521	-2.69608	2.69564	-0.86	2.10
FR-L17_R1	4.51420	-4.51406	4.51413	0.14	1.93
FR-V2_R1	-28.53627	28.53682	-28.53655	0.55	2.13
V2_R1-R2p	-20.96758	20.96843	-20.96801	0.85	2.22
V2_R2p-R3	14.16018	-14.16102	14.16060	-0.84	1.75
Суми:	-28.13426	28.13410	-28.13418	-0.16	

4.2 QUALITY CONTROL OF THE MEASUREMENTS

Levelling results are checked before further processing:

- By calculating the values for the height differences measured back and forth;
- By creating differences from the height differences between the front and back levelling;
- By comparing with the criteria defined for the realization of the measurements.[5]

The basic criteria of the accuracy for performing the measurements are defined through:

Allowed deviation of the dual height difference (forward-backward):

$$\Delta_{\Delta H}[mm] \leq 1.5\sqrt{S_{km}}[mm] \qquad (16)$$

s - length of the levelling line in km.

Allowed deviation of the sum of differences *d* from all levelling distances for the whole levelling line:

$$[d]\,[mm] \leq 2.25\sqrt{L_{km}}[mm] \qquad (17)$$

L - length of whole levelling line in km.

From the analysis of the difference between the height differences in the levelling sides (forward-back) and the allowed deviations, it can be noted that all the height differences fulfill the condition of accuracy.

4.3 ESTIMATION OF THE ACCURACY OF THE LEVELLING LINE

Mean error for 1 km levelling distance

The value **m = 0.26 mm/km** is determined by the differences d between height differences in forward and backward levelling in *mm*, from the levelling distances between benchmarks S in *km* and the number of levelling distances in the line between height differences in forward and backward levelling n_s by the formula [2] :

$$m = \pm\frac{1}{2}\sqrt{\frac{1}{n_s}\left[\frac{d^2}{S}\right]} \leq 0.40\,[mm] \qquad (18)$$

The mean systematic error for 1 km levelling distance

To calculate the systematic part of the total error from the levelling, the differences from the forward and backward levelling and the corresponding lengths of the levelling sides were used.

In formula (19) the length of the line is denoted by L and is in kilometers. L' is the length of a part the line that is roughly with the same constant impact of systematic errors. μ is the difference from the ordinates of the endpoints of the reggresion line, defined as approximation of d for part from the line, characterized by approximately the same influence of systematic errors expressed in mm. [2]

$$\sigma = \frac{1}{2}\sqrt{\frac{1}{L}\left[\frac{\mu\mu}{L'}\right]}\,[mm] \qquad (19)$$

When determining the regression line, its length must met the following requirement: the differences between the ordinates of this line and the graphical values of d should not exceed 4 mm. At the same time: the sum of the areas between the graph of the d and the regression lines on both sides to be equal. The value for is **0.15mm/km**. [2]

The mean random error for 1 km levelling distance

$$\eta = \frac{1}{2}\sqrt{\frac{[dd]}{L} - \frac{[ss]}{L^2}\left[\frac{\mu\mu}{L'}\right]}\,[mm] \qquad (20)$$

The value for η is also **0.15mm/km**.

Linear regression analysis

In addition are shown graphs of regression lines and partial plots of them, characterized by systematic errors with approximately constant impact. [2]

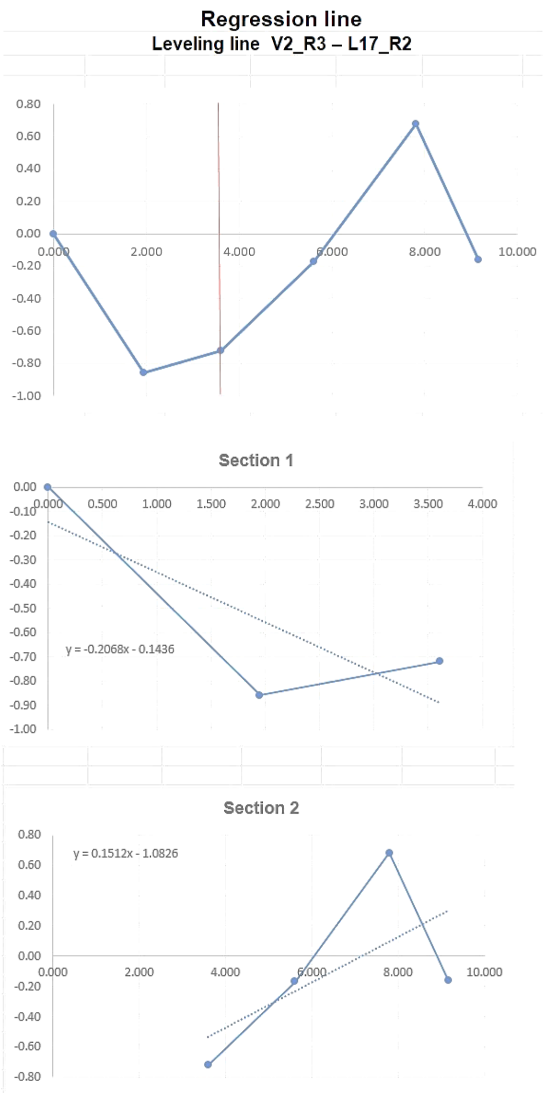

Figure 2. Regression lines and partial plots with similar systematic errors

4.4 TRANSFORMATION OF HEIGHT DIFFERENCES IN PHYSICAL DEFINED HEIGHT SYSTEMS

After the corrections with the mean meter of the pair of rods, the transformation of the height differences in physically defined systems of dynamic, orthometric, and normal heights was performed. For the transformation we use the equations shown in Chapter 2, using the normal value of the Earth's gravity for the coordinates of the central point of the Republic of Macedonia to the ellipsoid. The central point is determined on the basis of the coordinates of the state border of the Republic of Macedonia. For the latitude of the center point, the value B = 41.710193231336 (degrees, decimal) is adopted. [1]

The coordinates of the benchmarks and the values of the acceleration of the force of the earth gravity used in calculating the corrections for the transformation into physical systems are given in Table 3.

The approximate values of the heights of the benchmarks which are necessary for calculating the transformation correction in the physical systems of height are determined on the basis of the results of the levelling and the orthometric height of the FRSK benchmark from the second levelling of high accuracy of the Socialist Federal Republic of Yugoslavia: $H^{o}_{FRSK} = 251.90796\,\mathrm{m}$. [1]

Table 3. Registry of coordinates in ETRS89 and gravity data

Point	COORDINATES ETRS 89			g	mg
	B	L	h		
FRSK	41°59'59.89346"	21°25'55.97294"	296.486	980245.4349	5.6
L17-1	41°59'15.81895"	21°26'10.69754"	291.211	980248.2603	10.8
L17-2	41°58'18.50588"	21°26'41.53896"	288.744	980256.9443	15.9
V2-1	42° 00' 45.92706"	21° 25' 37.46418"	324.889	980240.5345	13.5
V2-2	42° 01' 17.94714"	21° 24' 33.93758"	304.3702	980246.1553	17
V2-3	42° 01' 57.62833"	21° 24' 11.31832"	318.2858	980247.7962	20.8

All data in Table 4 are given in meters, except for the Geopotential numbers and the auxiliary values CpVR given in m^2/s^2.

Table 4. Corrections and height differences in various physically defined height systems

From	To	dh-sm	Cp	DC	OC	NC	CpVR	DVR	OVR	OVR
Line:	L17					f = I + II				
FRSK	L17-R1	-4.51413	-39.7224	0.000349	-0.00068	0.00028	-44.235	-4.51378	-4.51481	-3.51481
L17-R1	L17-R2	-2.69564	-23.6862	0.000192	-0.00217	0.000417	-26.377	-2.69545	-2.69781	-1.69781

From	To	dh-sm	Cp	DC	OC	NC	CpVR	DVR	OVR	OVR
Line:	V2					f = I + II				
FRSK	V2-R1	28.53655	251.1433	-0.00233	0.00109	-0.00105	279.6744	28.53888	28.53546	28.53759
V2-R1	V2-R2	-20.96801	-176.339	0.00163	-0.00136	0.00021	-196.372	-20.9696	-20.96664	-20.96822
V2-R2	V2-R3	14.16060	116.4137	-0.00102	-0.00056	-0.00045	129.6388	14.16162	14.16116	14.16105

The tags used in table 4 are as follows:

Dh-sm	Height difference corrected for the mean meter of the pair of rods
Cp	Geopotential number
DC	Dynamic correction
OC	Orthometric correction
NC	Normal correction
CpVR	Auxiliary value
DVR	Height difference in the system of dynamic heights
OVR	Height difference in the system of orthometric heights
NVR	Height difference in the system of normal heights

5. CONCLUSIONS

By applying the dynamic, orthometric and normal correction and the other corrections explained in this paper, we can obtain a "best estimate" of observed, dynamic, orthometric or normal elevation differences. These corrections are applied to observed levelling data to minimize the effects of known systematic errors. And corrections applied for transformation in different physically defined height systems are for eliminating the effect of the nonparallelism of equipotential surfaces.

The data processing itself includes:

- First of all, the quality control of the measurements which was made in this paper and it was concluded that all levelling measurements were performed in accordance with the defined criteria for control and monitoring of the measurements;
- Estimation of the accuracy of the levelling line with mean error for 1 km levelling distance **m=0.26mm/km** and **0.15mm/km** for both systematic and random errors for 1 km levelling distance;
- Also graphs are shown of regression lines and partial plots of them, characterized by systematic errors with approximately constant impact;
- Next, the results of the levelling measurements were corrected with geometric corrections the mean meter of the pair of rods for uniform rod scale;
- Than transformation is performed of measured height differences in different physically defined height systems, with computing and applying dynamic, orthometric and normal correction.

The results of the data processing of the precise levelling measurements uniquely shows the need of applying the above mentioned corrections.

REFERENCES

[1] Agency for Real Estate Cadastre (2016), Report for data processing from the levelling network of high accuracy of Republic of Macedonia, Skopje.

[2] Agency for Real Estate Cadastre (2019), Report for data processing of precise levelling measurements of the three levelling lines (border links) between Bulgaria and Macedonia.

[3] D. Zekov, Gospodinov S. (1989)**,** За отчитане на рефракцията при високоточна геометрична нивелация, *Геодезия, картография и кадастар*.

[4] Emery I. Balazs, Gary M. Young, Corrections applied by the national geodetic survey to precise levelling observations, *National Geodetic Survey.*

[5] F. Kasapovski, et al., (2018): Vertical crustal movements in seismic active regions, *SJCE, Vol. 7 Issue 2*, Skopje.

[6] G.V Marinov, D.G. Ganceva, (2017), A study on the impact of the vertical refraction on the results of precise levelling measrements, *SJCE, Vol. 6 Issue 2*, Skopje.

[7] Jaakko Makinen, (2008) The treatment of the permanent tide in EUREF products, *Finnish Geodetic Institute*.

[8] Odalovic O. (2011), *Levelling Network of Macedonia* , Agency for Real Estate Cadastre, Skopje.

[9] Srbinoski Z. (2008), *Physical Geodesy*, Faculty of Civil Engineering, Skopje.

[10] Srbinoski Z., et al., (2017),Geodetic projects as part of main project for new basic levelling network, *SJCE, Vol. 6 Issue 2*, Skopje.