RESPONSE SURFACE METHOD ANALYSIS OF ULTIMATE CAPACITY OF INTELLIGENT COMPOSITE ANCHORING ELEMENT

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The article deals with analysis of transfer mechanism of the force from composite bolt to rock surroundings. The ultimate capacity of anchoring member depends on relatively wide range of input parameters. Moreover, the range of parameters of rock mass changes. The axisymmetric FEM of problem was constructed in Plaxis2D. The Mohr-Coulomb failure criterion, described by equivalent rock strength parameters determined by fitting an average linear relationship to the curve representing Hoek-Brown failure criterion, was used. Later, statistical analysis based on the design of experiment concept and the Response Surface Method (RSM) was carried out. The result of the full-factorial design and RSM analysis provide the regression model. It describes dependence of the bolt ultimate capacity Fy and corresponding deformation uy on the uniaxial compressive strength and Rock Quality Designation RQD. Results show that RQD starts to have significant impact on the Fy from the level of σc above approx. 80 MPa. The deformation uy is affected by the RQD conversely.

Keywords: Fiber Reinforced Polymer, Design of Experiments, Response Surface Method, Pull-out resistance.

1. INTRODUCTION

The amphibolite rock slope of height 10 m was considered in analysis, where the bolt is planned to be used as the anchoring member of passive flexible stabilizing system anchored to the ground, preventing the falling of rock to the railway. Stabilizing systems are formed by membranes, made of cable nets or wire meshes and bolts anchored to the ground (Blanco-Fernandez, 2011). For both types of system, the proper anchoring of their components to the ground is crucial. Different kind of steel rebars is commonly used as tendon of rock bolts. However, steel is susceptible to the corrosion and oxidation and it is not possible to prevent corrosion in long term. Further, steel rebar´s penchant to conduct electrical fields, which makes it undesirable in application for applications near to the railways with electrically powered vehicles. The reparation or replacement of damaged members is very expensive and problematic. Therefore fiber-reinforced polymer (FRP) rebars can serve as a good alternative to the steel rebars. FRP rebar is the composite material made of the fibers oriented in one direction, which are in thermoset polymer matrices. It won´t rust or corrode, it is also immune to road salt and it is inherently nonconductive, so it won´t interfere with the operation nearby electrical devices. The research aimed on development of the "intelligent" anchoring element (which will be able to monitor changes in its axial strain strain induced e. g. falling stones stopped by wire mesh etc. and immediately inform the responsible office about this change) has been launched in 2016 in Czech Republic. One part of the research is aimed on analysis of the pull-out capacity of those elements. The use concept of Response surface method (RSM) was adopted in this study to illustrate the effect of known factors, that affect the result. The analyzed process of pulling-out the bolt of ground was modelled using the Finite Element Method (FEM). Concept of combination FEM and RSM was used for geotechnical problem previously e. g. by (Wong, 1985) for the slope stability analysis or later by (Lin, 2016) to predict the facing deformations of geosynthetic-reinforced wall.

2. METHODS

2.1 FINITE ELEMENT MODEL OF EXPERIMENT

The Plaxis software 2D has been employed for FEM modelling of the pullout resistance for non-prestressed FRP rock bolt with diameter 20 mm inserted in the gravity grouted borehole of 30 mm diameter. The bond length of bolt was 1.0 m and the whole length is inserted into the cement grout. The bolt has been modelled vertically positioned to achieve the condition of axisymetry. The mesh of 2D 15-noded triangular finite elements with fourth order interpolation of displacement ad twelve Gauss points for the numerical integration has been employed along the embedded length of the bolt. Displacement controlled loading at the anchor head has been adopted. The contact between the anchoring element body and surrounding rock have been modelled by the interface finite elements, which are implemented in Plaxis R_{inter} = 0,9.

2.2 MATERIAL MODELS AND INPUT PARAMETERS OF FRP TENDON AND ROCK MASS

The GFRP tendon of bolt was modelled using linear elastic model with modulus of elasticity of FRP material $E_{f,m}$ = 50 GPa. Poisson´s ratio of FRP tendon was considered 0,2 according to (Mustafa, 2017). Strength and deformation characteristics of the tendon are summarized in the Table 1. The grout body was modelled by the linear elastic model with the value of E_{28} = 15 GPa.

The Mohr-Coulomb (M-C) material model was chosen for simulation of rock behaviour. The M-C strength parameters have been obtained by balanced fitting the Hoek-Brown (H-B) failure criterion by the M-C linear failure line. Material constant m_b of H-B criterion is a reduced value of the intact rock parameter m_i, which also depends on the Geological Strength Index (GSI) (Hoek, 2002) and the Disturbance Factor (D). The value of D was estimated D = 0.7 according (Hoek, 2012). Input parameters are summarized in Table 2. The rock quality, or else core recovery parameter, was evaluated by determination the Rock quality designation (RQD) according (Dere, 1967), which is a rough measure of the degree of jointing or fracture in a rock mass, measured as a percentage of the drill core in lengths of 10 cm or more. The value of Geological strength index (GSI) was then estimated according the correlation presented by (Hoek, 2013):

$$GSI = \frac{52\dfrac{J_r}{J_a}}{\left(1+\dfrac{J_r}{J_a}\right)} + \frac{RQD}{2} \qquad (1)$$

This relationship is based on the above mentioned RQD and on quotient J_r/J_a included in the Tunnelling Quality Index Q (Barton, 1974). This quotient represents the roughness and frictional characteristics of the joint walls or fillings. The balanced fit was then done by fitting an average linear relationship to the curve representing H-B failure criterion for a range of minor principal stress values defined by $-\sigma_t < \sigma_3 < -\sigma_3$, max. This led to the derivation of M-C equivalent effective strength parameters φ´ and c´. The closed form solutions for both the Generalized H-B and the M-C criteria have been used by (Hoek, 2002) to generate hundreds of solutions and to find the value of $\sigma'_{3,max}$, which determination is crucial to conduct above mentioned fitting. For the case of slopes, using Bishop´s circular failure analysis for a wide range of slope geometries and rock mass properties this analysis gave:

$$\frac{\sigma'_{3max}}{\sigma_{cm}} = 0,72\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0,91} \tag{2}$$

where σ_{cm} is the global rock mass strength. The value of Young´s modulus of intact rock E = 107 GPa (Lambe, 1969) was reduced in calculation according to (Hoek, 2006) taking the D and GSI into account, in order obtain a representative stiffness of the in-situ rock mass. Poisson´s ratio of intact rock was estimated by value 0.29. It is in accordance with (Lambe, 1969), where it is reported in range 0.28 – 0.30.

Table 1. Properties of GFRP tendon

Tensile strength (mean value)	$f_{t,m}$	1100	MPa
Tensile strength (characteristic value)	$f_{t,k}$	1050	MPa
Modulus of elasticity	$E_{f,,m}$	50	GPa
Poisson´s ratio	v	0.2	-
Density	ρ	2100	kg/m^3
Nominal diameter	$d_{,nom}$	18	mm
Diameter with adhesion layer	d	20	mm

Table 2. Properties of rock from geological survey

Density of intact rock	γ	2869	kg/m^3
Joint alteration and clay fillings	J_a	0.75	-
Joint roughness factor	J_r	2	-
Uniaxial compressive strength	σ_{ci}	36÷92	MPa
Rock Quality Designation	RQD 0÷1m	57	%
	RQD 1÷2m	36	%
	RQD 2÷3m	24	%

2.3 DESIGN OF EXPERIMENTS AND RESPONSE SURFACE METHODOLOGY

Design of Experiments (DOE) was used to process the results of previously described FEM calculations. This methodology is based on fractional or full factorial experiments, in which the studied design variables are altered at different levels in a systematic way, while the statistical significance of analysis is ensured. It can be seen an alternative way to full-probabilistic design (in the sense of consider randomness of input parameters into design) in the case, where the full information about the probability distribution of input parameters is not available and only few point realizations are known. The term experiment is defined in this concept as the systematic procedure carried out under controlled conditions to illustrate the effect of known factors, that affect the result. In presented study the experiment was conducted numerically. Factors, also called inputs, can be generally classified as either controllable or uncontrollable variables of analysed process. Hence, factors are predictor variables, also called independent variables, which are changed in systematically way during experiment to determine their effect on the response (also called dependent or output, variable) (Box, 2005). The influence of two factors RQD and σ_c on the response has been analysed. The ultimate carrying capacity F_y and corresponding displacement of bolt head u_y were considered as response. Both two factors were referred to as low, intermediate and high level. Three-level full factorial design 3^k was written. It means that *k* factors are considered, each at three levels. The experiment design matrix is written in Table 3.

3. RESULTS

The design pattern of three-level two-factor full factorial design is shown in the Table 3, where the treatment combinations are in the standard order. Randomizing the standard order is not necessary, because the experimental variability does not appear in the numerical experiment. Replication of treatment was also not considered for the same reason. The two last columns of Table 3 are added to the pattern and contain the calculated responses. The linear model with interactions describing the dependence between predictors and response was evaluated. The relationships (3) and (4) represent the resulting regression functions describing analysed dependence.

Table 3. Experiment design matrix with the resulting F_y and u_y

Run Order	Pt Type	Blocks	RQD [%]	σ_c [MPa]	F_y [kN]	u_y [mm]
1	1	1	57	36	29	0,6
2	1	1	57	62	30	0,8
3	1	1	24	36	26	0,9
4	1	1	57	92	44	1,0
5	1	1	36	36	24	0,7
6	1	1	36	62	36	0,9
7	1	1	36	92	43	1,1
8	1	1	24	62	30	0,9
9	1	1	24	92	34	1,0

Diagnostical parameters, that serve for evaluation the efficiency of regression analysis, are summarized in the Table 4 for both models.

Table 4. Estimated regression coefficients for Fy and uy

Term	F_y			u_y		
	SE_coef	T	P	SE_coef	T	P
Constatnt	10,1767	1,963	0,107	0,1678	6,141	0,002
RQD	0,2463	-0,305	0,773	0,0041	-2,979	0,031
σ_c	0,1511	0,883	0,418	0,0025	-0,077	0,941
RQD* σ_c	0,0037	0,821	0,449	0,0006	2,112	0,088

The contour plots and also the 3D surface plots were created to explore the relationship between three variables (two independent factors RQD and σ_c) and the response F_y (Figure 1) and u_y (Figure 2). Contour plots display the 3-dimensional relationship in two dimensions, with x and y factors (predictors) plotted on the x and y scales and response values represented by contours. A contour plot can be seen as a topographical map in which x, y and z values are plotted instead of longitude, latitude and elevation.

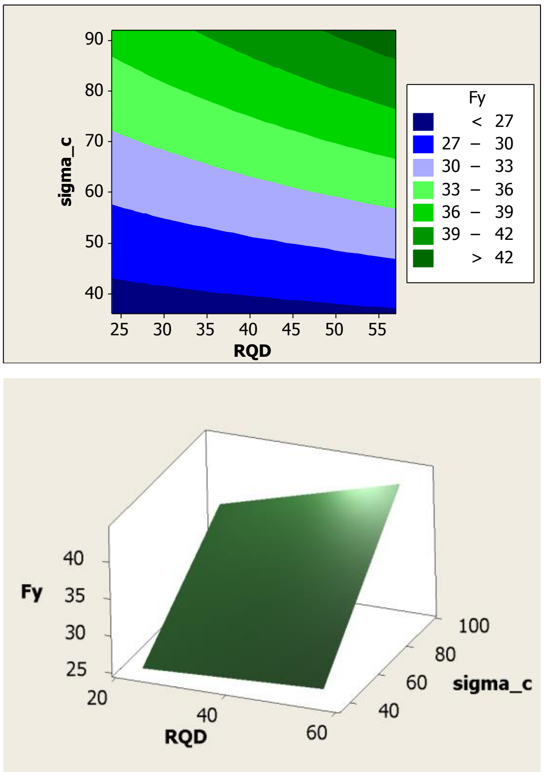

Figure 1. Contour and Surface plot of the ultimate carrying capacity F_y vs σ_c; RQD

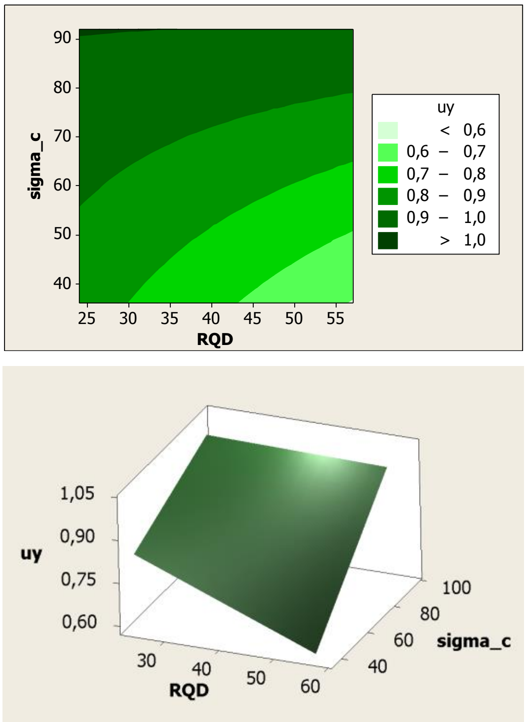

Figure 2. Contour and Surface plot of the deformation measured at the bolt head u_y vs σ_c; RQD

4. DISCUSSION AND CONCLUSIONS

The mathematical-statistical models have been formulated using RSM, in which two factors with three levels were implemented. RQD and σ_c has been chosen as factors, and consequently as the main input parameters of FEM model of rock bolt. The ultimate carrying capacity F_y and corresponding displacement uy of bolt head were followed up output responses. The statistical package MINITAB was used to design and analyze the results of numerically (FEM) calculated experiments. The following conclusions and recommendations can be drawn from the analysis:

• The available geological survey characterizing the rock by determination the RQD values was available. Although there was relatively lack amount of information in the survey applicable directly as input parameters for numerical modelling, there are possibilities how to calculate them cautiously using available correlations. The RQD values was thus transformed to the GSI, which serves as the input for the H-B constitutive model. As the exhausting formulation of the calculation model of bolt was not the task of study, the H-B failure criterion was balanced by the M-C failure line in the range of stresses expected in the analysed slope. The M-C material model with equivalent strength parameters was consequently used for simulation of rock behaviour.

• The linear statistical model with interactions, describing the dependence between predictors and response, was evaluated after finishing all runs of FEM calculation. The statistical significance of the coefficients evaluated on the significance level α = 0,05, are summarized in Table 4. The regression was more successful in the case of the model of for uy, where the lower p-values were achieved. It should be noted that in case of F_y the coefficients were statistically insignificant on the chosen level α. The percentage of variation $R\text{-}S_q$ in the response that is explained by the model was $R\text{-}S_q(F_y)$ = 84,75 % and $S_q(u_y)$ = 90,33 %. The use of higher order models (linear + squares and full quadratic) was analysed also, but it led to the over-fitting the model. It was accompanied by the lower $R\text{-}S_q$, which represents the percentage of variation in the response that was explained by the model. The statistical significance of squared combination of predictors was lower, which was indicating by higher p-values calculated in ANOVA. The full potential of DOE methodology cannot be utilized in experiments based on numerical simulations (e. g. some statistical tests exploring experimental variability) because the experimental variability does not appear in this type of simulations. Although this fact, it still has a significant contribution to effective designing and conducting experiments that leads to a reduction the number of tests needed.

• The results of analysis prove the expected qualitative estimate: the bigger values of RQD and σ_c, the bigger ultimate forces F_y are achieved. The contour and surface plots graphically illustrate those results. Above that, it can be deduced from those plots, that the RQD starts to have higher impact on the ultimate force Fy from the level of σ_c approx. 80 MPa. The deformation uy is affected by the RQD conversely (RQD has bigger impact on results below the level of σc approx. 80 MPa). The numerical results gave deeper insight into the quantitative impact of every considered factor on the monitored response. The ultimate force and corresponding deformation of the bolt is calculated in the form of the range of interval. It is more realistic result, especially in the case of weathered rock massifs, than taking the single deterministically designed value into account.

• Regardless of simple constitutive models, this initial study confirmed the usability of the DOE and RSM concept for the examination and illustration of the effect of factors that affect the response of calculation. Their usefulness will be more obvious, when the more advanced material models and more factors (e. g. different bond lengths or pressure grouting) will be included in the mathematical models. The three-level design was used in case of this study. It is prohibitive in terms of the number of runs, and thus in terms of cost and effort. For example, the two-level central composite design with centre points is much less expensive in case of more considered factors, while it is still a very good way to establish the presence or absence of curvature in case of more factors affecting the response, or in case of more time-consuming calculations. When the proper calibration of the appropriate FEM model will be done, e.g. via the comparison with the results of the full-scale tests that are planned in next phases of the research project, the stronger statistically significant regression relationships can be derived by the RSM concept. Those can then serve as the kind of design formulas for the design of resistance of analysed bolts.

Acknowledgements

This research was financially supported by the research project No. FV10505 of Ministry of Industry and Trade of the Czech Republic and project LO1408 AdMaS UP of Ministry of Education, Youth and Sport of the Czech republic - Programme of Sustainability I.

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