Elena Popovska
MSc, Civil Engineer
e-popovska@hotmail.com

Mile Partikov
PhD, Assistant Professor
University “Ss. Cyril and Methodius”
Faculty of Civil Engineering – Skopje
partikov@gf.ukim.edu.mk

Denis Popovski
PhD, Associate Professor
University “Ss. Cyril and Methodius”
Faculty of Civil Engineering – Skopje
popovski@gf.ukim.edu.mk

STIFFNESS COMPARISON OF UNSTIFFENED AND STIFFENED T - JOINTS OF HOLLOW SECTIONS

Structural analysis is performed on 12 models of T-joints from hollow cross sections reinforced with welded flange plate on the chord. Cross sections used for the verticals are sized from 40 to 70mm square section while the chord has size of 100mm square hollow section. For each vertical three different thickness are analyzed, also considering the thickness of the chord is same as the thickness of the vertical in the respective model. Joints are loaded with maximal in-plane bending moment. The results are drawn in a form of moment – rotation diagram and table result and compared to results obtained for same unstiffened models. Reinforcement used is based on the assumption that all the models comply with chord face failure mode, which is expected for the geometry used for these types of joints. It is concluded that the type of reinforcement is properly chosen since in all models there is an increase of the initial stiffness. Additionally, is summarized that the increase of the stiffness relies greatly of the solidness of the used sections.

Keywords: stiffness analysis, T-joints, moment connections, reinforced connections, hollow sections.

1. INTRODUCTION

The behavior of a Vierendeel beam markedly relies on the stiffness of the joints. This type of beam is consisted exclusively of T-joints that are loaded with a combination of a bending moment, shear force and certain axial force. Since it is very complex to do and properly interpretate separate analysis of such joint when all three internal forces are present in the joint usually is recommended studies to be done of joint loaded with in-plane bending moment as a the most influential force. According to stiffness classification joints can either be found as pins, semi rigid or rigid and while rigids are rarely used due to economic reasons, mostly we are talking either about pins or semi-rigids. According to the Eurocodes few parameters are highly important for the definition of a T-joint geometry. Factor ‘β’ which is ratio of the mean width of the brace members, to that of the chord: factor ‘γ’ which is the ratio of the chord width to twice its wall thickness and factor ‘β_p’ which is the ratio of the width of the brace members, to that of the stiffening plate. Expressions for these factors point out the limits where joints change classification, as well define the mode of failure. For models loaded with in – plane bending moment two modes of failure are expected: if $\beta \leq 0{,}85$ plastic failure of the chord face is applicable, while when $\beta > 0{,}85$ failure of the side wall of the chord by yielding, crushing or instability is expected.

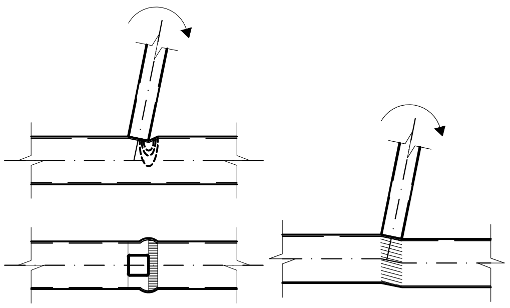

Figure 1. Chord plastification mode of failure on the left; chord side wall failure mode on the right

Another factor to distinguish pins and semi rigids is the value of ‘γ’ where bigger values lead to thicker chords and stiffer joints.

Provided the fact that the Vierendeel is greatly depending on the stiffness of its joints pinned joint in this type of member will lead to increase in the internal forces of the chord and considerable increase of the global deformation, which not always will be bearable. So, when increase of a cross section is not an option next step is to select proper type of reinforcement. This also relies on the mode of failure.

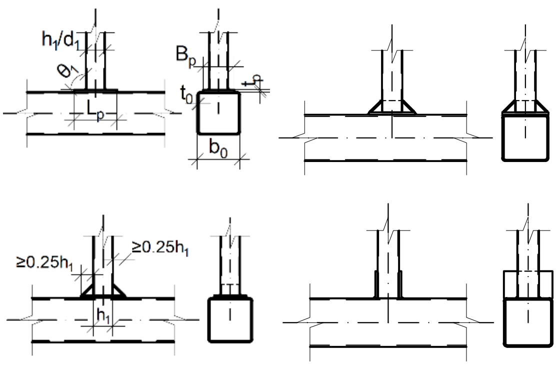

Figure 2. Recommended types of reinforcement in joints where chord plastification failure is expected

Reinforcements when a failure due to plastification of the face of the chord is expected can be in a form of haunch welded between the chord and the vertical, welded ribs aligned with the webs of the vertical to the chord, angles positioned in between the vertical and the chord to avoid direct welding between the two elements, and most used plate welded to chord’s face.

All these reinforcements will secure this weak spot in the joint and move the failure to another section by also increasing the general stiffness.

Another important factor when obtaining stiffness of a joint is to perform correct classification. Every geometry of joint has values of limit where pins end and limit where rigids start. Everything in between is semi rigid. These values can be pointed on a diagram of moment - rotation. The diagrams are obtained in an iterative calculation with incremental increase of the moment to obtain the representative rotation and the curve of the correlation M-φ. Also, from the value of the stiffness of the connection, value of the initial stiffness of the joint is derived.

This research focuses on the analysis of T-joints classified as pins, whose geometrical factor $\beta \leq 0{,}85$ and factor $2\gamma > 16$. Reinforcement is performed by welding a stiffening plate $\beta_p < 0{,}85$ on the face with of the chord what is according to the recommendations of Eurocode 1998, chapter 8, annex E table 7.17. The behavior of these models is compared to the behavior of the same unstiffened models and conclusions about the factors that influence the stiffness are drawn.

2. NUMERICAL ANALYSIS

Analysis is performed in software IDEA StatiCa where two separate calculations are performed. First a design capacity analysis is done for all models to obtain the maximal value of the bending moment and next the main stiffness analysis is run with this bending moment. With these more accurate values for the secant stiffness are obtained.

2.1 SOFTWARE THEORETICAL BACKGROUND

Software IDEA StatiCa is designed for calculation of steel joints (connections). Four types of analysis can be performed: stress/ strain analysis where plates and connectors are checked, stiffness analysis, design resistance analysis, and capacity analysis. The calculations are based on the component based philosophy where this method (CM) solves the joint as a system of interconnected items – components. Each component is checked separately using corresponding formula derived from the codes. To avoid the generality of the component-based method the internal forces in the components are calculated by FEA. Elastoplastic behavior with hardening of the material is considered. The welds are designed as a multipoint constraint that relate the finite element nodes of one plate edge to another. This way of modeling is conservative and leads to the fact that the resistance of the weld along the length will rely on the stress peaks that appear at the end of plate edges, in corners and rounding. To eliminate these effects, a special elastoplastic element is added between the plates that redistributes the stress peaks along the length of the weld and real values are obtained.

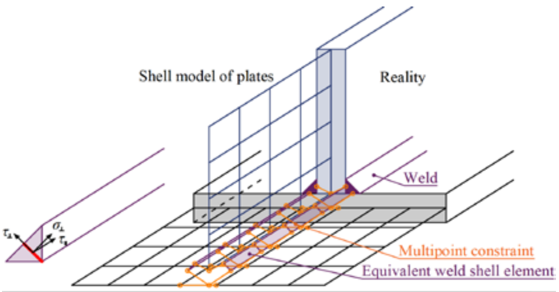

Figure 3. Multipoint constraint weld model

2.2 MATHEMATICAL MODELS

The geometry of each model is given in the table 1. Material used for all elements is S275JR. The welds of each model are one-sided filed weld with thickness same as the thickness of the plates they connect. The cross sections used are cold formed square hollow sections.

Table 1. Models geometry

Model	chord	vertical	reinforcement
1.1	[]100.100.3	[]40.40.3	≠90.6…110
1.2	[]100.100.3	[]50.50.3	≠90.6…110
1.3	[]100.100.3	[]60.60.3	≠90.6…115
1.4	[]100.100.3	[]70.70.3	≠90.6…115
2.1	[]100.100.4	[]40.40.4	≠90.8…110
2.2	[]100.100.4	[]50.50.4	≠90.8…110
2.3	[]100.100.4	[]60.60.4	≠90.8…115
2.4	[]100.100.4	[]70.70.4	≠90.8…115
3.1	[]100.100.5	[]40.40.5	≠90.10…110
3.2	[]100.100.5	[]50.50.5	≠90.10…110
3.3	[]100.100.5	[]60.60.5	≠90.10…115
3.4	[]100.100.5	[]70.70.5	≠90.10…115

In the software the appropriate cross sections are selected. For the chord length of 1000mm is used in all models while the vertical is 400mm. The connection between the elements is done by using predesigned layouts where values for the geometrical characteristics of the stiffening plate and welds are specified, and relations of the connecting plates with welds are assigned. In the design resistance analysis bending moment of 1kN is applied on the vertical and the analysis is started. The analysis finishes with a factor which multiplied by 1kN will give the value of the maximal bending moment the joint can bear. This is done separately for each model.

These moments are next used in the main stiffness analysis. For this analysis the member of the vertical is assigned as analyzed member, on which the bending moment is going to be applied. For the software to do a classification of the joint also the theoretical buckling length should be inputted. The chord is considered as continues beam with two supported endings while the vertical is loaded on one ending and connected on the other.

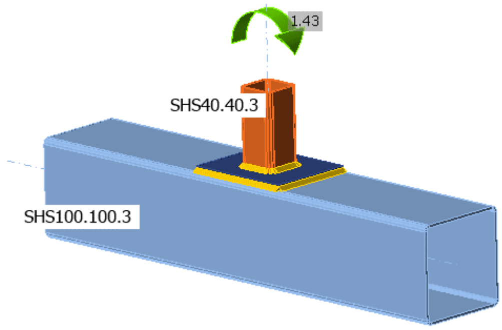

Figure 4. 3D model of model 1.1.

The stiffness analysis is run, and results are requested in both diagrams and table representation.

2.3 RESULTS AND DISCUSSIONS

From the analysis diagrams of the curve moment - rotation correlation is drawn. Apart from this, information can be got about the plastic moment. To obtain this curve the program applies load steps and evaluates the rotation of the connection. Additional information can be gotten about joints ultimate resistant moment $M_{j,Rd}$, the ultimate plastic moment of the cross section of the analyzed member or the vertical $M_{c,Rd}$, value of the initial stiffness $S_{j,ini}$, value of the secant stiffness $S_{j,s}$, rotation deformation and rotational capacity φ and $φ_c$, boundary where rigids start $S_{j,R}$, boundary where pins end $S_{j,P}$ and class.

The model 3.1 with vertical []40.40.5 displayed biggest difference of the stiffness between stiffened and unstiffened model while smallest difference happened in model 1.4 with vertical []70.70.3. Models of vertical with width 40mm display nonlinear rapid increase of the initial stiffness after reinforcement. Similar behavior, display models of vertical with width 50mm, while the ones of wider verticals display linear increase by the increase of the stiffness. Their lines are almost parallel. Same applies to the thickness of the models, in the models with thickness of 3mm the increase is somewhat same, but there is great difference in the stiffness after reinforcing the models of 5mm. All models after reinforcement change class from pins to semi-rigid. The model 3.1 with vertical []40.40.5 is closest to the boundary of rigids. By comparing the diagrams of unstiffened and stiffened joint can be assumed that the reinforced joints display smaller changes in the rotation. Also, the angle between the horizontal and the M-φ curve is bigger compared to the unstiffened ones.

Table 2. Stiffness difference after applying reinforcement

Model	Stiffness increase	Increase in %
1.1	6.55	655%
1.2	4.64	464%
1.3	3.82	382%
1.4	2.81	281%
2.1	10.06	1006%
2.2	6.27	627%
2.3	4.97	497%
2.4	3.85	385%
3.1	22.87	2287%
3.2	9.36	936%
3.3	5.89	589%
3.4	4.97	497%

Table 3.Class boundaries and initial stiffness

Model	**Sj,P [kNm/rad]**	**Sj,R** [kNm/rad]	**Sj,ini** [kNm/rad]
1.1	24,47	1223,25	87,20
1.2	51,19	2559,38	126,39
1.3	92,14	4606,88	106,89
1.4	150,94	7546,88	333,23
2.1	29,14	1456,88	281,59
2.2	62,21	3110,63	308,18
2.3	114,45	5722,50	435,64
2.4	189,26	9463,13	672,09
3.1	35,18	1758,75	1083,16
3.2	70,88	3543,75	756,14
3.3	132,56	6628,13	912,22
3.4	222,08	11103,75	1288,65

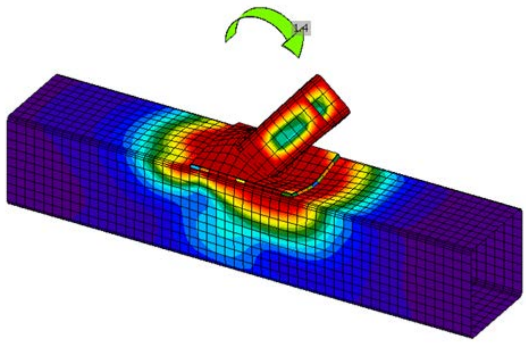

Figure 5. Results of loaded model 1.1
[]40.40.3 - []100.100.3 + ≠90.6…110

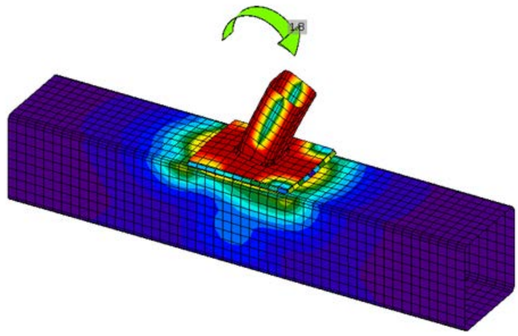

Figure 6. Results of loaded model 1.2
[]40.40.4 - []100.100.4 + ≠90.6…110

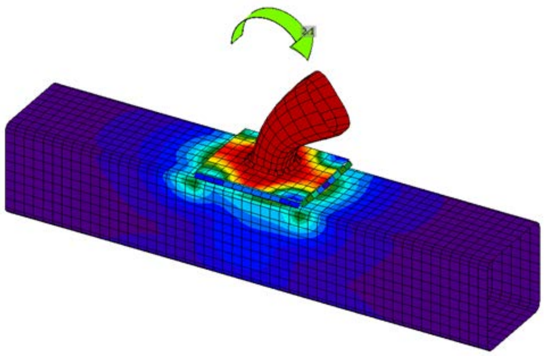

Figure 7. Results of loaded model 1.3
[]40.40.5 - []100.100.5 + ≠90.6…110

Table 4: Results of calculated stiffness of model []40.40.3 - []100.100.3 + ≠90.6…110

Name	Comp.	Loads	Mj,Rd [kNm]	Sj,ini [kNm/rad]	Φc [rad]	L [m]	Sj,R [kNm/rad]	Sj,P [kNm/rad]	Class.
SHS40.40.3	My	LE1	1.44	87.20	0.08	0.40	1223.25	24.47	Semi-rigid

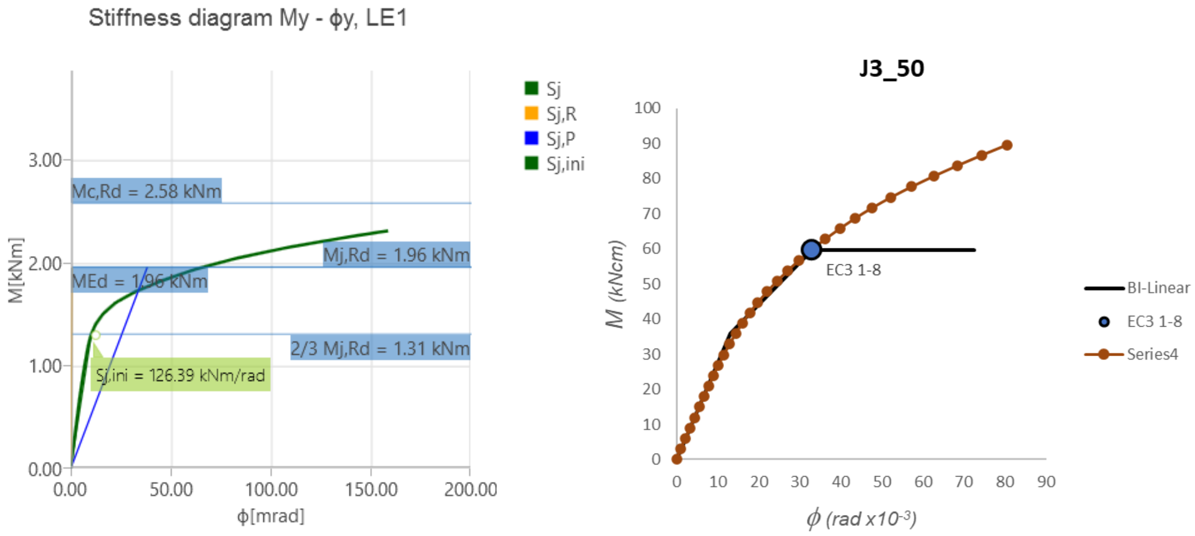

Figure 8. Moment rotation curve of a reinforced joint on the left; moment- rotation curve of the initial joint

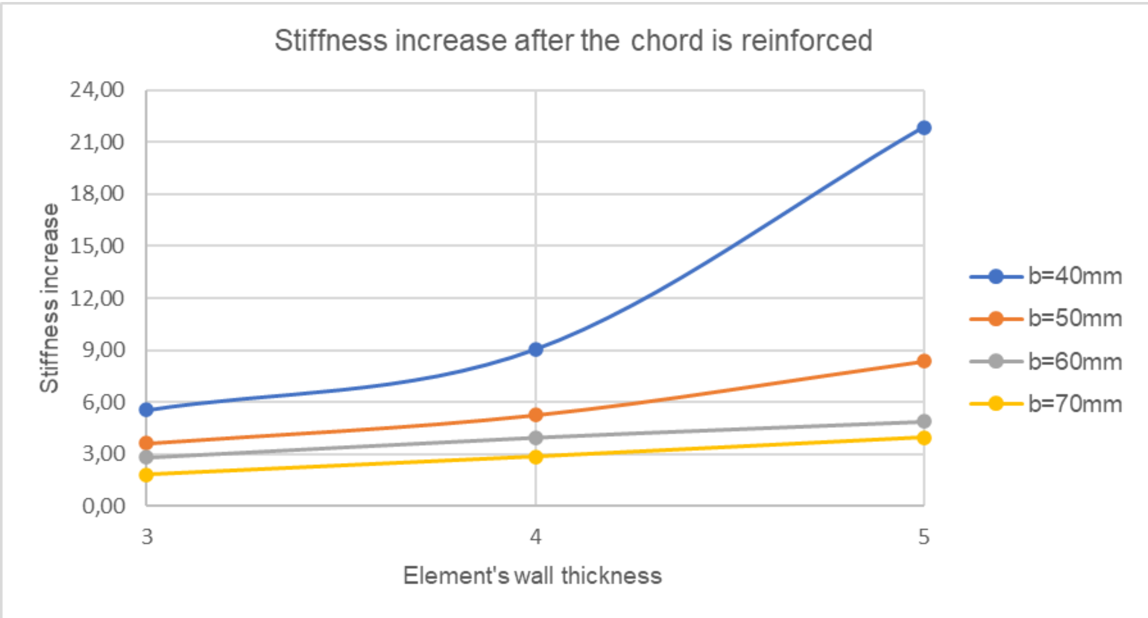

Figure 9. Diagram of the increase of the stiffnesses to vertical geometry

4. CONCLUSION

Based on the numerical results the following can be concluded:

(1) All joints after reinforcement increase their stiffness leading to conclusion that the initial assumption of failure due to plastification of the face of the chord is correct. This increase varies from 2,81 to 22,87 times the initial stiffness. Additionally, all models change class from pin to semi rigid, meaning that while the reinforcement does help in the stiffness none of the models reaches rigid class. This indicates that other factors considerably influence the stiffness of a T-joint. Model 3.1 with a vertical of a SHS40.40.5 is closest to the boundary with an initial stiffness of 1083kNm/rad in comparison to the 1758kNm/rad where rigids start. Also, the initial stiffness is 60% of the value where rigids start, while in model 1.4 with a vertical of a SHS70.70.3 the initial stiffness is 4% of the value for rigids limit.

(2) Reinforcement has a great benefit in the models with thicker walls and

smaller size vertical. Models 1.1, 1.2 and 1.3 all with a vertical of 3mm have almost similar increase in the stiffness from 6,55 in the smallest vertical to 2,81 times in the largest vertical. Contrary to this the models with thick verticals have increase starting from 4,27 to 22,87 times the initial stiffness. Taking all these factors in account leads to a conclusion that the effects of thin-walled elements which persist after reinforcement of the chord will influence the general stiffness of the global system.

(3) Models with large sized verticals don't have rapid increase of the stiffness like the small sized. This can be because these models have β ratio closer to the boundary of 0,85 which indicates that although their primary mode of failure is plastification of the chord they are also influenced of the side wall failure specific for models with β ratio bigger than 0,85.

(4) T-joints with smaller sized verticals depict more drastic increase of the stiffness with the increase of their wall thickness, while this is not the case with the joints with wider verticals. On the joined diagrams for the stiffness it can be seen that the models 1.1, 2.1 and 3.1 all with a vertical of a SHS40.40.3 have a rapid increase in the stiffness especially the leap between model 2.1 and 3.1 where from 9 times it will reach 22 times in the next one. This is also happening in models 1.2, 2.2 and 3.2 but with lower impact where the increase starts with 4,6 to 6,27 and ends with 9,36 times increase. In the other models the increase is almost linear. It can be concluded that an increase of 1mm thickness makes significant difference of the ratio width to thickness of the vertical which leads to thicker cross sections, while this increase is not that remarkable in the big sized cross sections of the vertical bracing. This also confirms the theory that the stiffness is greatly influenced by the thin-walled effects of the cross sections used.

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